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ABSTRACT

Exponential growth is a mega-concept which has many applications. It is fundamental to understanding how and why systems work and fail, be they natural or man-made systems. Lessons/activities in this booklet are designed for Florida teachers to help provide their students with an understanding of the nature and implications of exponential growth. Lessons/activities may be used in such diverse subject areas social science, sciences, home economics, English, and mathematics. The booklet is divided into four parts. Background information for teachers is presented in part one. Activities designed to foster understanding of the nature of exponential growth and to provide opportunities to reflect upon the consequences of exponential growth in the realms of energy and environmental consumption are provided in sections two and four respectively. Objectives and instructional strategies are included for activities/projects in these sections. Part three presents a list of 31 activities designed to help students develop insights into patterns of arithmetic/geometric growth. Most are relatively ambiguous in their assignment format to stimulate students to utilize their personal life experiences to connect to exponential concepts. "Forgotten fundamentals of the energy crisis," a paper by Albert A. Bartlett, is included in an appendix. (Author/JN)

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101 WAYS TO TEACH ABOUT EXPONENTIAL GROWTH
AND
ITS CONSEQUENCES

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Tri-County Teacher Education Center
WAYNE H. DUNCAN, Director
426 School Street
SEBRING, FLORIDA 33870

101 WAYS TO TEACH ABOUT EXPONENTIAL GROWTH

AND

ITS CONSEQUENCES

Tri-County Teacher Education Center
Wayne H. Duncan, Director
426 School Street
Sebring, FL 33870

Editor:

Dr. Rodney F. Allen, Florida State University
Tallahassee, Florida

AUTHORS:

Mr. William Hammond, Director of Environmental Education and Instructional Development Services, Lee County School System Fort Myers, Florida

Mr. Tom Baird, Project Manager, Pasco Energy Management Center, Port Richey, Florida

Dr. Roger Messenger, Director of the Center for Energy Conservation, Florida Atlantic University, Boca Raton, Florida

Dr. Don E. Marietta, Jr., Professor and Chairperson for the Philosophy Department at Florida Atlantic University

Dr. David E. Lahart, Energy Education Consultant for the Florida Solar Energy Center, Cape Canaveral, Florida

Mr. Richard Farmer, Coordinator of Math and Science for the Highlands County School System, Sebring, Florida

Mr. R. Frank Kapp, Consultant for Environmental Education, State Department of Education, Tallahassee, Florida

Ms. Phyllis Snyder, Science Teacher at Lake Placid High School, Lake Placid, Florida

Mr. Noble Littell, Mathematics Teacher at Lake Placid Middle School, Lake Placid, Florida

Mr. Lee Fowler, Mathematics Teacher at Lake Placid Middle School, Lake Placid, Florida

Dr. Rodney F. Allen, Professor in the Department of Curriculum and Instruction at Florida State University, Tallahassee, Florida

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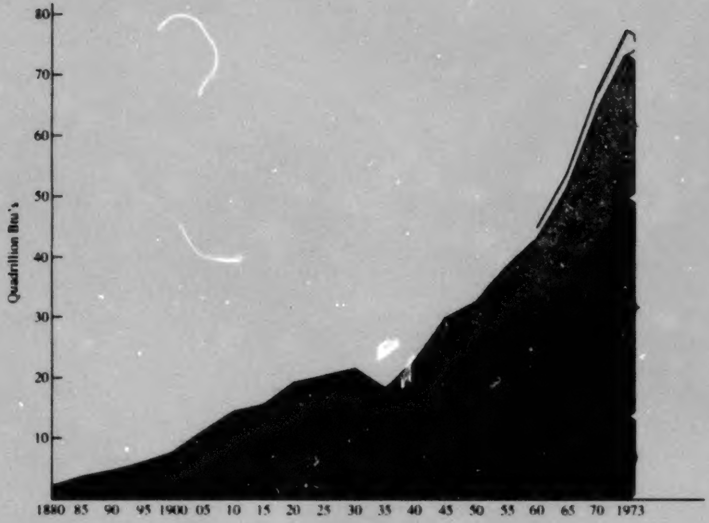

Energy Consumption in the U.S. by Sources, 1880–1973

INTRODUCTION

The American popular vocabulary gained a new phrase in the mid 1970s--energy shortage. It should not have been a surprise. The signs of its coming had been plain for several years, but most Americans and their media ignored them.

First, there were troubling shortages of natural gas that led to curtailments for low priority users, such as industries and electric utilities. Then, there was the pinch in oil supplies in the winter of 1972-73 followed by spot shortages of gasoline across the nation in the summer of 1973.

These, however, were scattered occurances with only limited impact on our public and its leaders. It took the Arab oil embargo of October, 1973, to drive home the crucial lesson once and for all. The lesson was this: The United States of America was grossly dependent on oil and natural gas to maintain its material standard of living. Because domestic oil and gas reserves were and are declining and energy demand was increasing geometrically, the U.S. had long ago become an energy importing nation and was dependent upon foreign petroleum sources.

If we had examined the facts of our situation and the exponential growth characteristics of per capita energy increases, it would not have taken an embargo to alert us to our problem. Similarly, today, there are other exponential impacts upon resources and our environment--without any embargo to warn us--which will have the same disasterous consequences in the future, unless we understand exponential growth and its implications.

This lesson booklet is produced for teachers in Florida to help provide an understanding of exponential growth to their students who will face these consequences--unless action is taken. The lessons may be used in diverse subject areas--social science, sciences, home economics, VoTec, English, etc.

Exponential growth is a mega-concept which has many applications. It is a fundamental concept to understand how and why systems work and fail--be they natural or human-made systems. We would urge teachers to use these lessons and use them well--for what we know as Florida may well be dependent upon citizens and leaders applying such concepts to public policy and personal behavior!

--Wayne H. Duncan, Director
Tri-County Teacher Education Center

--C. Richard Tillis, Director
Florida Office of Energy and
Environmental Education

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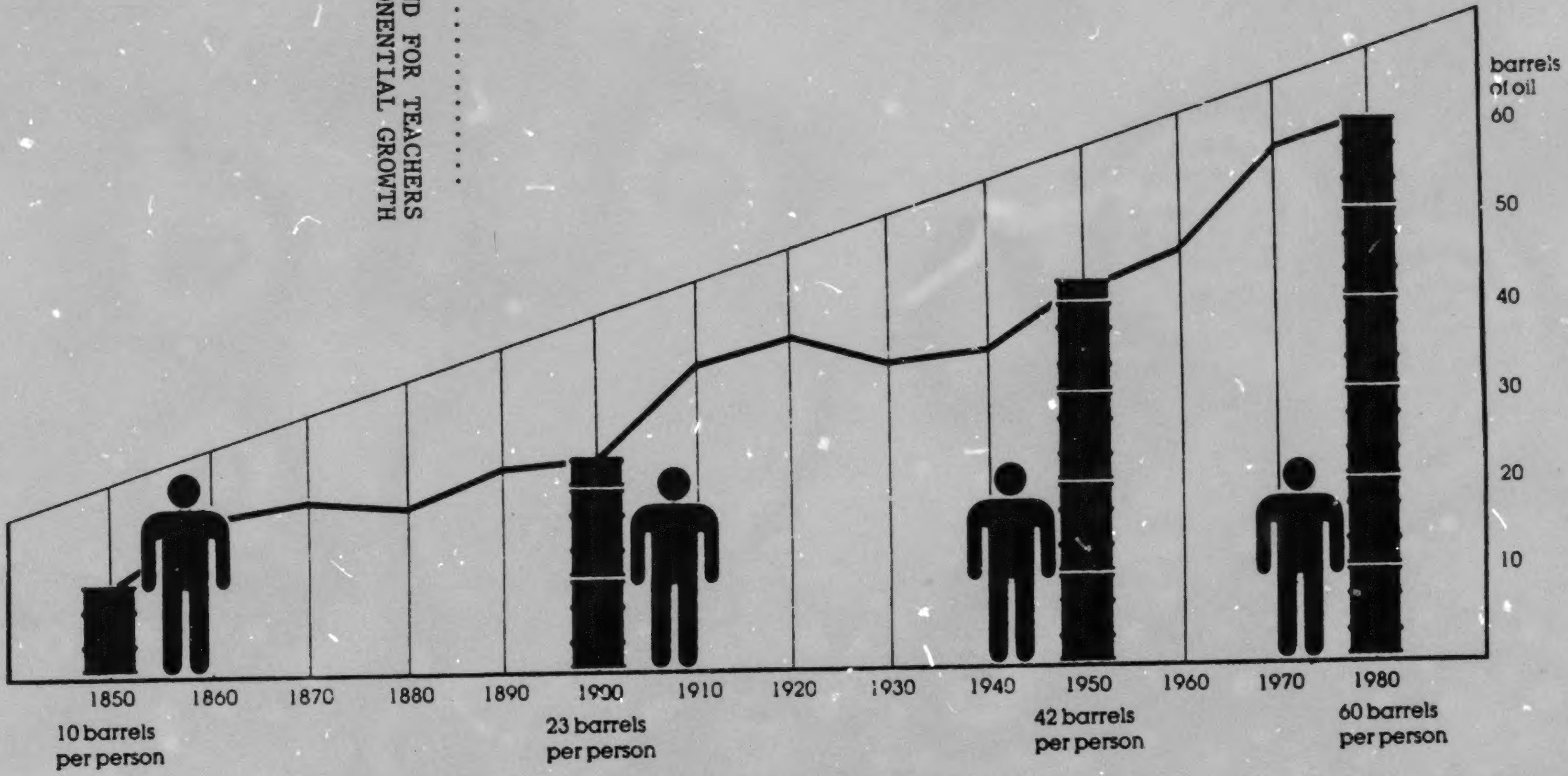
PART ONE..............
BACKGROUND FOR TEACHERS
ON EXPONENTIAL GROWTH



Growth in U.S. Annual per Capita Energy Consumption

The high stakes of teaching exponential growth

We gamble with our energy future when we fail to take into account the implications of exponential growth in resource consumption.

Joseph C. Cotham

"Double or nothing," the gambler's traditional challenge, is clearly understood by most people as offering risky, but definite, possibilities of reward.

However, the concept of doubling as related to quantities that grow at some *constant* annual rate—like compound interest, for example—is not at all widely understood. In fact, physics professor Albert Bartlett, who has written widely on projected growth of energy consumption, is convinced that the greatest shortcoming of the human race is its inability to understand increases of this kind—increases characterized by exponential growth.

Consider a contemporary version of an apocryphal tale that mathematicians have traditionally used to demonstrate the dramatic nature of exponential growth. It is the story of a monarch who, moved to magnanimity by the recent discovery of petroleum deposits within his kingdom, decided to offer his oil minister what he considered reasonable gifts—land, tankers, and a portion of projected oil revenues. But the minister demurred, and, pointing to the chessboard that had provided the king with much needed diversion during his long reign, suggested that the king give him instead an apparently modest gift. He requested one tiny thimbleful of oil (about one milliliter) for the first square of the chessboard, two for the second, and so on, with each successive square representing double the amount of the previous square. The king agreed and then ordered his treasurer to compute the size of the gift. Much to his horror he discovered that he had promised his minister over one hundred trillion barrels of oil—more oil than exists in the entire world! Whatever the king's ultimate response to his minister's chicanery, he had undoubtedly learned a convincing lesson on the effects of doubling.

What is not generally understood is that the phenomenon of doubling occurs whenever a quantity (whether representing production or consumption) grows at a *fixed percent* per year. For example, the rate of inflation in the U.S. for 1979 was 11.5 percent, which is to say that prices at the beginning of 1980 were 11.5 percent higher than they had been one year before. Were this inflation rate to continue, then each year's prices would exceed those of the previous year by 11.5 percent. Since the doubling time for an 11.5 percent rate of growth is about 6 years, by the end of 1985 prices would have *doubled* over what they were in 1979.

There is a simple relationship between doubling time and annual rate of growth, or:

$$\text{Doubling time} = \frac{70}{r}$$

That is, doubling time is approximately equal to 70 divided by the annual percentage rate of growth. Thus, a 10 percent rate of growth results in a doubling time of 7 years, and a 5 percent rate in a doubling time of 14 years.

The relationship between rate of growth and doubling time is explicit because growth at a fixed percent per year is defined by a mathematical function: the exponential function. This function relates the growing quantity to the original amount multiplied by a factor that increases geometrically as time increases. Growth of such diverse entities as savings accounts, population, and resource consumption is described using the exponential function.

Examining consequences

An examination of energy consumption patterns will illustrate the striking consequences of exponential growth that underlie Bartlett's concern.

In 1979 we in the U.S. produced about three billion barrels of crude oil out of an estimated total reserve of 182 billion barrels. Thus, were production to remain constant, we would run out of domestic oil in 61 years. Consider, however, how rapidly reserves disappear when a modest degree of growth in production is included in our calculations—say, 4 percent per year. This annual rate of growth seems at least realistic because it describes the growth in production from 1930 to 1970. The question is, of course, how long can increased production continue at this rate? A 4 percent growth rate would double the amount of oil produced in 17.5 years (70 ÷ 4), resulting in the disappearance of reserves in less than two doublings—or 31 years. Thus, exponential growth in consumption can result in much more rapid disappearance of a resource than would be the case if it remained at some constant level.

Of course, this example is a great simplification of a complex issue. Estimates of petroleum reserves are notoriously imprecise. New reserves, such as those in Alaska's North Slope, may be discovered. And increases in the cost of petroleum may cause the annual rate of growth to diminish. Yet, the implications of doubling remain the same. Even a doubling of current estimates of petroleum reserves would add, at a constant 4 percent rate of growth, less than one more doubling period—only 13 years of additional consumption. The lesson is stark: When consumption is rising exponentially, even a doubling of the remaining resource results in only a small increase in its life expectancy.

The inability of domestic oil production to keep pace with consumption (reflected most ominously in our increasing dependence on imported oil—45 percent of total consumption for 1979) has turned the attention of policy makers to alternative fuels. Coal, in particular, is being hailed as a major energy alternative for the future. The recently passed Energy Security Act, designed to spur the development of synthetic fuels, provides for increasing our utilization of coal as a feedstock for synthetic fuel production. But in order to meet the production goal of two million barrels of oil

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Joseph C. Cotham is a visiting assistant professor, Department of Science and Environmental Education, Indiana University, Bloomington, IN 47405.

equivalent by 1992, a lot more coal (in addition to oil shale) is going to have to come out of the ground than is mined today.

Enthusiasm for coal blossomed after the Arab oil embargo, resulting at times in wildly optimistic projections about coal's prospects in the future. The U.S. has been called the Persian Gulf of coal, with estimates of the duration of coal reserves ranging as high as 500 to 800 years.

How do these claims measure up to the inexorable implications of doubling and exponential growth? Current coal production is 776 million tons per year, and an optimistic but realizable rate of growth in production is 5 percent per year,[1] which results in a doubling every 14 years. Even though estimates of total recoverable reserves are highly variable, we may choose 1500 billion tons as a reasonable estimate. With that as a limit, a 5 percent growth rate allows fewer than 7 doublings—or a resource lifetime of 91 years.

How many gamblers have made a pilgrimage to the meccas of Nevada, secure in their naive understanding of a doubling strategy—"whatever the loss, double your wager until you win"—only to discover how rapidly their resources actually disappear.

Are we similarly naive in gambling with our energy future? Is our economy so committed to the imperatives of growth that we will not comprehend the implication of exponential growth in resource consumption until the coffer is nearly empty? The seriousness of these implications for the American lifestyle suggest that we would be well advised to incorporate an understanding of energy and exponential growth into our lifestyle choices and policy decisions—on the double! ■

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[1] Coal consumption has varied so enormously in this century that this exercise should be viewed primarily as one way of testing the implications of various assumptions.

References

1. Bartlett, A.A. "Forgotten Fundamentals of the Energy Crisis." *Journal of Geological Education* 28:4-35, 1980.
2. Bartlett, A.A. "The Exponential Function" (article series). *The Physics Teacher* 14:393-401, 485; 15:37-38, 98, 225-226; 16:23-24, 92-93, 158-159; 17:23-24, 1976-1979.

Principles of Exponential Growth

* Growth is said to be exponential when a quantity such as the rate of consumption of a resource (measured in tons per year or barrels per year) is growing at a fixed percent per year.

* With exponential growth, the growing quantity will double in size (increase by 100%) after a specific time:

$$T_2 = \frac{70}{P}$$

T_2 = Doubling Time

P = The constant percentage of growth

* Exponential growth is characterized by doubling, and after a few doublings can lead quickly to enormous numbers.

* The increase in any doubling is approximately equal to the sum of all the preceding growth. When the rate of consumption is growing a mere 7% per year, the consumption will double in 10 years--and the total consumption during that 10 years exceeds the total of all the previous consumption in history of that quantity!

A. When consumption is rising exponentially, a doubling of the remaining source results only in a small increase in the life expectancy of the resource.

B. When we are dealing with exponential growth, we do not need to have a precise estimate ot the size of a resource in order to make reliable estimates of how long the resource will last.

* The amount of the resource available (being consumed exponentially) is relatively insignificant compared to the rate of use (consumption increases).

Not all of the issues connected with exponential growth are technological or scientific in nature. Questions of motivation and ethical concern are crucial. Just teaching the hard facts of growth may not result in behaviors which are most likely to preserve human life for many generations to come and to protect the natural environment from escalating degradation. We must be able to deal with attitudes which accept the facts and deny their implications.

Why Worry?

One attitude may be expressed this way: I know all this looks pretty bad on the surface, but you are overlooking something--human ingenuity. The same technology which produced these problems will solve them. When we run out of one thing, we'll find a substitute. We'll build artificial environments. We'll build space stations. Have a little faith and hope! Hasn't human ingenuity always found an answer?

Teachers must face this attitude squarely. Here are some important considerations.

1. Humans have not always found an answer to the technological problems of their societies. The demise of great civilizations, the deserts lying in once fertile areas, and the poverty of once rich lands attest to the fact that technological fixes are not always found or are not discovered in time to save people from the consequences of their abuses of nature.

2. The technological problems we now face are far more complicated and of vaster scope than ever before. The problems which humans have solved in the past were within the confines of natural biological processes and did not get very far ahead of nature's own clock. With the use of fossil fuels in the industrial age, of non-biodegradable materials in the age of chemical technology, and of poisons with thousand-year half-lives in the atomic age, we face problems which are qualitatively different from those of any other time. Just as the student who can add and subtract may fail calculus, our technological genius may not be able to solve our problems in the time available.

3. Technological answers to problems tend to produce other problems, often unforeseen problems. Nature is such a complex web that we cannot do just one thing. Who know what new problems will be created by our efforts to solve our present problem with technological fixes.

4. It is possible that a new affordable fuel source might be discovered and that our other problems might be solved, but we have no definite idea how the solving of this complex of problems associated with exponential growth could be solved. How much risk can we afford to take? A good poker player does not bet on filling an inside straight. Even if we must gamble with human life, let's be smart about it. The stakes of exponential growth are not high enough to justify reckless risk taking.

Why Care?

One attitude toward the consequences of exponential growth might be, "Why should I care? The benfits come first, during my lifetime. The harsh penalties come later, after I'm gone. Now we can have better cars, more pizza, places and nightclubs, neat clothes and lots of travel. Why should I care about the eventual hunger, disease, and gasping for breath?"

To face up to this attitude we need to think clearly about ethical issues. We need to think about human beings and their place on this planet. We need to consider whay they have a right to do to the planet, and what they must not do.

What are human beings? What should they try to be?

Aldo Leopold, in his Sand County Almanac, said that humans must extend their concept of ethics to include the natural environment. Ethics has grown over the centuries, and it must grow some more. Our sense of responsibility must grow to include responsibility toward the land itself. Leopold said that humans must stop trying to be conquerers of the biosphere (he called it "the land community"), and become plain members and common citizens of this community of plants, animals, and the soil which provides for the life of all.

What are we? We are animals. We were created by the same process as produced all other species. We are injured by the absence of essential resources and are remarkably similar to other forms of life, and no wonder--they are our kin!

What should we do? What sort of behavior is fitting for animals like us? Is it fitting for us to destroy the other species and the sources of our common life? Or is it fitting for us to preserve ourselves and our fellow creatures? Is it fitting for us to preserve ourselves and our fellow creatures? Is it fitting for us to destroy, or to protect, the most beautiful planet we know of?

We need to think seriously about these things. We must learn to answer the why questions along with the what and how questions.

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PART TWO........................

THE IDEA OF EXPONENTIAL GROWTH

"TEASERS"

1. Get a job where you are paid 10¢ an hour, doubled every hour. How much will you be making in the 8th hour?
2. Would you rather work for 10¢ per hour doubled or for $7.35 per hour?
3. Try playing golf for an apple per hole, doubled with each hole. How many apples is the 18th hole worth?

A FOOLISH DAUGHTER?

OBJECTIVE: Students should be able to:

Solve a puzzle showing that exponential growth generates verylarge numbers in extraordinary short periods of time.

MATERIALS: Dittoed copies of the Handout: "A Foolish Daughter?"

PROCEDURE: Allow about ten minutes or so for the class to solve the puzzle. Then review the process used to solve it.

"A Foolish Daughter?"

The Daughter's Allowance:

Day 1	$ .01	Day 17	$ 655.36
Day 2	.02	Day 18	1,310.72
Day 3	.04	Day 19	2,621.44
Day 4	.08	Day 20	5,242.88
Day 5	.16	Day 21	10,485.76
Day 6	.32	Day 22	20,971.52
Day 7	.64	Day 23	41,943.04
Day 8	1.28	Day 24	83,886.08
Day 9	2.56	Day 25	167,772.16
Day 10	5.12	Day 26	335,544.32
Day 11	10.24	Day 27	671,088.64
Day 12	20.48	Day 28	1,342,177.28
Day 13	40.96	Day 29	2,684,354.56
Day 14	81.92	Day 30	5,368,709.12
Day 15	163.84	Day 31	10,737,418.24
Day 16	327.68		

NAME____ DATE__________

PUZZLE:

A Foolish Daughter?

A father complained that his daughter's allowance of $5 per week was too much. The daughter replied, "Okay, Dad. How about this? You give me a penny for the first day of the month, 2¢ for the next, 4¢ for the next, 8¢ for the next, and so on for every day of the month." The father, thinking he had a foolish daughter, readily consented.

Who, indeed, was the more clever? Use the space below for your answer.

A GRATEFUL KING'S REWARD

OBJECTIVE: Students should be able to:

Solve a puzzle that shows that exponential growth generates very large numbers in extraordinary short periods of time.

MATERIAL: Dittoed copies of the Handout: "A Grateful King's Reward."

PROCEDURE: As a follow-up activity, ask students to write riddles of their own, and try to "stump" the rest of the class.

"A Grateful King's Reward"

How much wheat does the "pauper" receive from the king? By following the pauper's instructions, you will have 2^{63} grains on the 64th square (the last square on the chessboard) and the total grains on the board will be one grain less than 2^{64}. How much wheat is 2^{64}? Simple arithmetic shows that it is approximately 500 times the current annual harvest of wheat in the ENTIRE WORLD. This amount is probably larger than all the wheat that has been harvested in the history of humankind. How did the pauper get this enormous number? He started with one grain of wheat and doubled it a mere 63 times.

Square 1 = 2^0 = 1 grain
Square 2 = 2^1 = 2 grains
Square 3 = 2^2 = 4 grains
Square 4 = 2^3 = 8 grains, etc.

Square 63 = 2^{62} =
4,628,070,013,058,678,784 grains

Square 64 = 2^{63} =
9,256,140,026,117,357,568 grains

TOTAL number of grains on the board:

9,256,140,026,117,357,568

NAME ______________________________ DATE __________

PUZZLE

A Grateful King's Reward

The king of an ancient country in an ancient time was bored. The royal dancers bored him. His children playing offered him no pleasure; even the achingly beautiful days bored him. In his desperation to relieve his boredom, he fired the court jester and offered a reward to anyone in the kingdom who could provide him with new entertainment. The reward would be granting anything the successful entertainer wanted.

Several months passed before a ragged pauper arrived at the castle with an offer to teach the king a new game--chess. The king liked this new game so much he played it every day. When it came time for the pauper to leave, the king asked him what he would like for his reward.

"Grains of wheat," said the pauper. When he saw the king's eyebrows rise in astonishment, he added, "If you will place but one grain of wheat on the first square of the chessboard, two on the second, four on the third square, and so, the amount being doubled each time, I will be content with the amount I receive."

The king protested. This was a poor reward for all the good things the pauper had done. But the king was also greedy. "It shall be as you wish," he said, lowering his eyes so the pauper would not see the twinkle of anticipation in his eyes. He clapped his hands twice for his servants and ordered them to bring him a sack of wheat.

The new game of chess began.

How much wheat will the pauper receive at the end of play on the sixty-fourth square? Use the space on the back for your answer.

SAVE THE LILY POND

OBJECTIVE: Students should be able to:

Solve a puzzle showing that exponential growth generates very large numbers in extraordinary short periods of time.

MATERIALS: Dittoed copies of the Handout: "Save the Lily Pond."

PROCEDURE: Allow about ten minutes or so for the class to solve the puzzle. Then review the processes for solving the puzzle.

"Save the Lily Pond"

The answer to the problem is the 29th day. On the 29th day the lily would cover half the pond. You have one day to save the pond.

An extension activity for "The Lily Pond": Distribute graph paper (4 squares to the inch is best). Have students mark off a big square, 32 small squares by 32 small squares (8" x 8"). This represents the pond.

If the lily plant would have completely filled this pond on the 30th day, how many of the small squares did it occupy on the 20th day? Have students guess before they begin to figure.

The simplest way of arriving at a solution is to work backwards, like a time-lapse movie run in reverse. First divide the pond in half and label one half 30. This is the portion that was added on the thirtieth day. Then divide the remaining half again and label one portion of that 29. Continue in the same fashion, dividing the unlabeled half of each segment in half again until you reach the number 21. The remaining portion will be single square, the total size of the plant on the 20th day.

NAME ______________________________ DATE __________

PUZZLE:

Save the Lily Pond

Suppose you own a pond on which a water lily is growing. The lily doubles in size each day. If the lily was allowed to grow unchecked, it would completely cover the pond in 30 days, choking off the other forms of life in the water. For a long time the lily plant seems small and you decide not to worry about cutting it back until it covers half the pond. On what day will that be? Show your calculation in the space below.

"CHEAP GASOLINE FOREVER?"

OBJECTIVE: Students should be able to:

Solve a puzzle showing that exponential growth generates very large numbers in extraordinary short periods of time.

MATERIALS: Dittoed copies of the Handout: "Cheap Gasoline Forever?"

PROCEDURE: Allow about ten minutes or so for the class to solve the puzzle. Then, review the answers and the processes for solving the puzzle.

"Cheap Gasoline Forever?"

1. about 10 years
2. about 13 years
3. about 17 years
4. probably in the first ten years or so you had yourself a deal. After that time, you would quickly decide that you could not possibly meet the price of gas.
5. $1,572.87

NAME_____________________________________DATE__________

PUZZLE:

Cheap Gasoline Forever?

Suppose you meet an oil company executive as you drive into the gas station to fill up your gas tank. He pulls you aside and says, "Have I got a deal for you! I will offer you a lifetime contract for gasoline, starting this year at only 1/10 cent per gallon. The terms of the contract state that the price will double every year, and that you must buy from me for the rest of your life." Does this sound like a good deal?

1. How many years will it be before you are paying today's prices for gasoline?

2. How many years before the price hits $10.00 per gallon?

3. How many years before the price hits $100.00 per gallon?

4. Was the price "a deal" after all? Explain.

5. You decide to buy a motorcycle in the year 2001. The gasoline tank holds 1.5 gallons. How much will it cost you to fill the tank?

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RICE

OBJECTIVE: To explore how consumption of energy coupled with exponential growth leads to depletion of energy resources.

MATERIALS: Checkerboard for each student
Five pounds of rice

BACKGROUND: To understand the implications of exponential growth in population, in resource consumption or in pollution levels, calculate the doubling time of a growing quantity. A handy rule of thumb is to divide the percent rate of such growth into the number 70 to arrive at the doubling time. For example, the population of the world, now roughly four billion persons, is growing at about 1.6 percent per year. The doubling time is 70 divided by 1.6, or 44 years. So long as that rate of growth holds constant, we can calculate the future population as follows:

4 billion people in 1975
* plus 44 years................8 billion people in 2019
* plus 44 years...............16 billion people in 2063

To say this another way, a child born in 1975 who lives 88 years will experience first hand what it is like in a world of 16 billion persons or will witness first hand what it was that intervened to forestall the 16 billion.

PROCEDURE:

1. Give each student a checkerboard, some rice and directions to place one grain of rice in the lower left hand square, two grains in the next square, four in the next, eight in the next, 16 in the next and continue this pattern. Have students estimate the amount of rice they will need.

   When students realize they have run out of room and are faced with an impossible task, discuss these questions in class.

   a. Were you able to fill all the squares? if not, why not?

   b. How is doubling of rice grains in the squares like our current consumption of energy?

   c. What will happen if we continue energy at our present rate of consumption?

2. Discuss the game of "Rice Checkers." There are 64 squares on the board. If the number of grains of rice is doubled 63 times, that will be 2 to the 63rd power grains of rice on the 64th square. They would have to put more rice in the last square than there is rice in the whole world. No one was able to complete the game because he ran out of room. The current consumption of energy is increasing at a rapid rate, too.

3. Show students how to calculate doubling time T_2, using the following expression:

$$T_2 = \frac{70}{\text{annual \% growth rate}}$$

4. Calculate T_2 for each of the following examples:

   a. Inflation (and electricity consumption) at 7% per year.

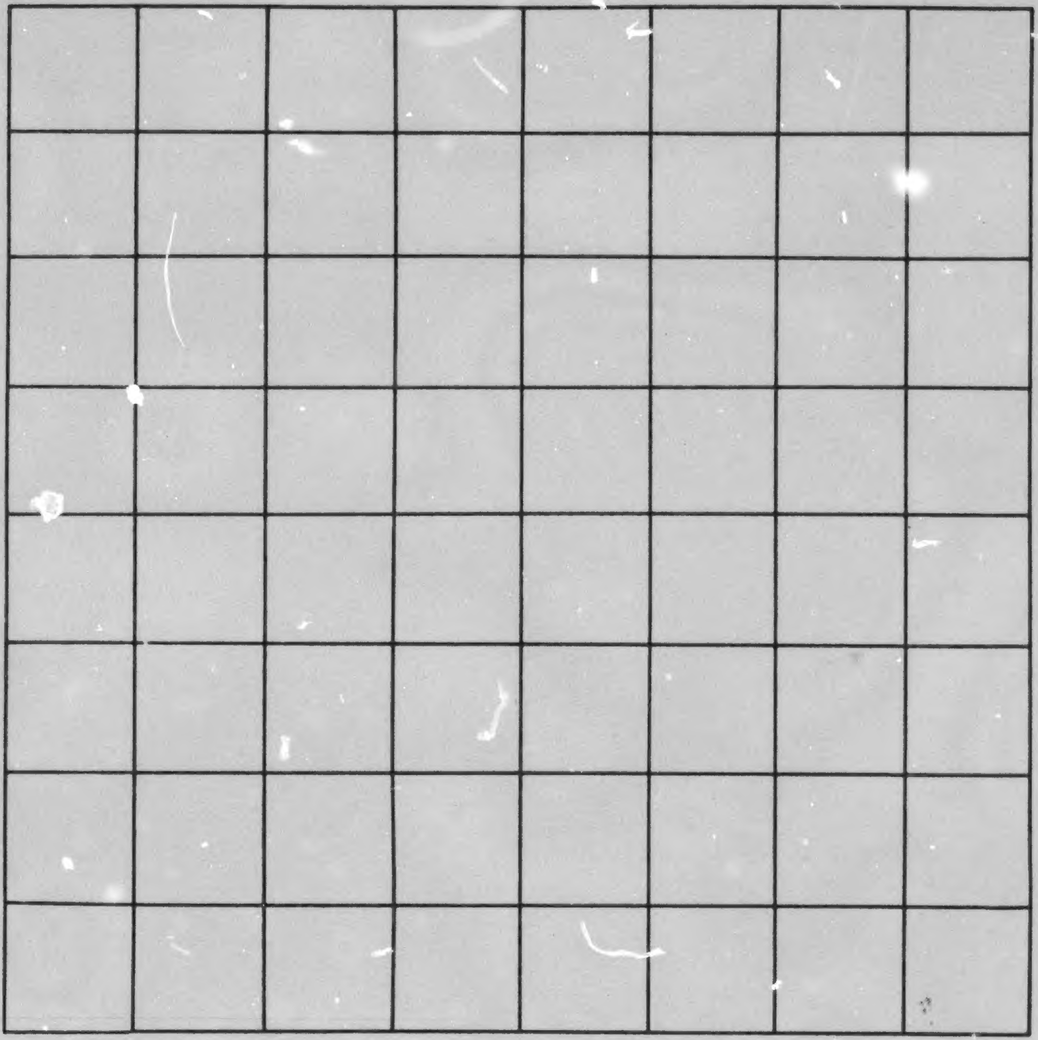
   b. Population at 1.64% per year.

   c. Energy consumption at 3% per year.

SOURCE: The Jefferson County Public Schools, Jefferson County, Colorado.

NAME____ DATE__________

RICE CHECKERBOARD



BACTERIA BOTTLES

OBJECTIVE: Students should be able to:

Solve a puzzle showing that exponential growth generates very large numbers in extraordinary short periods of time.

MATERIALS: Dittoed copies of the Handout: "Bacteria Bottles."

PROCEDURE: Allow about ten minutes for the class to solve the puzzle. Then, review the answers to the puzzle.

PUZZLE:

"Bacteria Bottles"

1. The bottle was half-full at 11:59 p.m. This is true because the doubling time is one minute and the bottle was full at midnight.

2. Answers will vary. To clarify, ask students: "At 11:55 p.m., when the bottle was only 3% filled and 97% empty, would it be easy to perceive that there was a space problem?

3. With space resources quadrupled, the bacterium have two more doubling times, or two minutes before they will run out of space.

11:58 p.m. - Bottle 1 is one quarter full.
11:59 p.m. - Bottle 1 is half-full.
12:00 Midnight - Bottle 1 is full.
12:01 a.m. - Bottles 1 and 2 are full.
12:02 a.m. - Bottles 1, 2, 3, and 4 are all full.

When things grow exponentially, enormous increases in resources are consumed in a very short time.

Multiplying by division could point out how vital it is to curb the growth of population and energy consumption. The discussion could expand to include possible ways to avoid the dilemma of the bacteria.

Question 2 raises the psychological issue of awareness of rapidly approaching danger while there appears to be plenty of space in which to grow. This should be stressed as dramatically as possible. Also, this story can help the teacher deal with the claim that environmentalists are "doom sayers," "prophets of doom," or "gloomy pessimists." It might be asked, "What would bacteria as smart as most humans say about a bacterium who cried "danger!" when the bottle was 1/8 full?"

Question 3 raises the issue of newly discovered resources. This story is ideal to put new discoveries in perspective. The point should be stressed that the amount of a resource is much less significant than the rate of use.

When using this story, attention should be given to the fact that in each doubling period as much is used as in all previous history.

NAME ______________________________ DATE ______

PUZZLE:

Bacteria Bottles

Bacteria multiply by division. One bacterium becomes two. Then two divide into four; the four divide to give eight, and so on. For a certain strain of bacteria the time for this division process is one minute. If you put one bacterium in a bottle at 11:00 p.m., by midnight the entire bottle will be full.

1. When would the bottle be half-full? How do you know?

2. Suppose you could be a bacterium in this bottle. At what time would you first realize that you were running out of space?

3. Suppose that at 11:58 some bacteria realize that they are running out of space in the bottle. So they launch a search for new bottles. They look far and wide. Finally, offshore in the Arctic Ocean, they find three new empty bottles. Great sighs of relief come from all the bacteria. The number is three times the number of bottles that they've known. Surely, they think, their space problems are over. Is that so? Explain why the bacteria are still in trouble.

   Since their space resources have quadrupled, how long can their growth continue?

4. Does what you have learned about bacteria suggest something about human population growth?

GROWING ALL THE TIME

1. Refer to the Biological Science Curriculum Study textbook, Green version, for some superb laboratory activities on growth. See:

   Lab. 12.3: Growth of Yeast Populations (pp. 379-380)

   Lab. 2.2: Hypothetical Study of Sparrows (pp. 42-44)

   These will teach students the typical growth curves and the semoid curve of natural populations. In addition, students are introduced to a semi-log graph.

2. Students' ears and muscles can demonstrate, in an immediate and physical way, exponential growth.

   PROCEDURE A: Play a note on the piano and start a stopwatch (or a pencil to a desk top).

   At the end of one minute strike the same note twice. After another minute play the note 4 times, after three minutes, 8 times, after 4 minutes - 16 times and so on.

   In a very short time the player's hand will be unable to move fast enough to sound the note.

   PROCEDURE B: Supply students with colored pencils, crayons, and graph paper.

   Students color: 1 square the first minute
   2 squares the second minute
   4 squares the third minute
   etc.

   After only 9 or 10 minutes doubling time is usually sufficient to exhaust both the students and the surface of the paper.

3. Growth is a common sign of health and vitality. Parents expect a normal infant, for example, to double her birth weight in about five months' time. If the baby doubled her weight again in the next five months, however, the parents would have a rather chubby testimonial to their well-meaning care and overnutrition. Should the baby continue to double her weight at five-month intervals, she would probably be too fat to walk at 15 months and obese at 20 months. Such an expansion is an example of exponential growth - and it's consequences.

Beginning with their actual birth weight, ask students to calculate what their own weight would presently be had they doubled their body weight every five months of their life.

4. An analogy of the Earth as an oil tanker is a graphic way of illustrating our resource depletion rates. Students and the general public often do not believe our energy problems are real, and that there are sufficient stores of new oil off Mexico, the North Sea, Alaska or our own continental shelf to last centuries at even higher rates of consumption than those we now experience.

   What if the entire Earth was simply a hollow sphere filled with oil? We wouldn't have any petroleum resource worries. Right?

   If Earth were a thin-walled hollow, spherical spaceship, it would have a "hold" 12,000 kilometers in diameter full of petroleum. This is the theoretical upper limit of fossil fuel our planet could contain.

   6.8×10^{21} barrels (trillions of billions). With maximum fuel storage and a spiralling U.S. rate of consumption during the last 100 years (an increase of 7 percent per year), the "Supertanker" would be entirely drained in about 330 years!

   Rather than a full hold, we have only a thin crust with spotty pockets of petroleum here and there -- and these are getting harder and harder to find.

Adapted from Simonis, Davis A. 1982. "Energy: The Analogy Approach," The Science Teacher 49(2): pp 41-44.

AAAS TEACHER NOTES

APRIL 1982

Insects can either be beneficial co-inhabitants of the planet Earth or one of humanity's most harmful nemeses. The Medfly can be categorized as one of the more persistent pests to infiltrate agricultural areas. In fact, the results of eradication efforts recently made in the San Francisco Bay Area may soon prove who is the winner in this confrontation between man and insect.

WANTED: Preferably Dead (or . . . Some Basic Facts about the Medfly)

Identifying Characteristics:

Size: About that of the housefly (5-6 mm)

Yellowish orange marks on drooping wings

Black spots on its back

Light gray bands on its yellowish abdomen

Can be present as adult, egg, larva, or pupa. Eggs and larvae most dangerous to fruit crops. Methods for control: Fruit Stripping, Pesticides, Sterile Male Flies, Quarantine, and Traps.

REWARD: The control of the Medfly could save California from a $15 billion loss to its economy.

Source: Hagen, *et al.*, "Mediterranean fruit fly: The worst may be yet to come," in *California Agriculture*, March-April, 1981, pp. 5-7

#1: Read "The Fruitless Pursuit?" in *Science 82*.

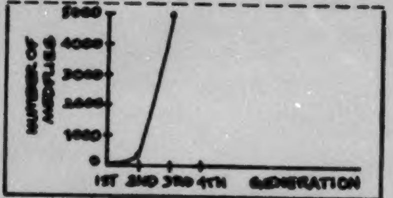
Assign this article to your students to read before beginning the other activities. Suggest that they also look up other information about the Medfly in the library, especially on the pros and cons of the various eradication methods.

#2: Graphing a Medfly Population

1. Give the students the following data:
   - A. One female is capable of laying 300-500 eggs a month.
   - B. Of these, 50 females and 50 males usually survive.
   - C. Each of the females is then capable of producing 50 more surviving females.
2. A pair of Medflies arrives in California via some non-inspected citrus fruit. If there is time for 3 more generations of Medflies to be reproduced during the summer, how many Medflies could this one female generate by summer's end? (Remember, on the average, one male Medfly survives for each female.)
3. Help the students graph this information, labeling the horizontal axis "Generation" and the vertical axis "Number of Medflies." Graph paper with 10 squares to the inch works best for this exercise, with one square equaling 100 Medflies. What problems develop in graphing the fourth generation?
4. Introduce the following variables into the population computations:
   - A. Suppose sterile male flies were introduced during the second generation and 25 sterile males mated with 25 females. What could this do to the third and fourth generations of Medflies on the graphs? Have the students graph their new data. Discuss when the best time would be to introduce sterile male Medflies into the "growing" season
   - B. Suppose spraying of Malathion took place during the second generation's pupal stage. What effect would this have on population growth? (Remember, Medflies live underground during their pupal stage.)
5. Remind the students that their resulting populations came from one male and one female. What would the total populations be if we began with 200 pairs? 1000 pairs?

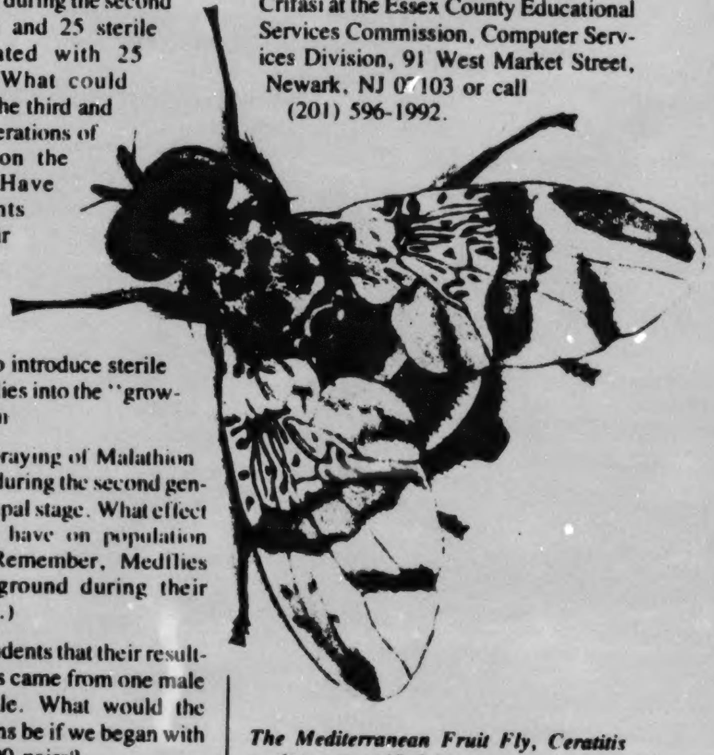
MEDFLY

"A Fruitless Pursuit?"



#3: STERL: A Computer Simulation about Insect Population Control

If you have a computer available for student use, you can obtain a computer program that will involve your students in a decision-making experience similar to that happening with Medfly eradication efforts in California. The program involves trying to control the growth of a population of screw worm flies with the methods of pesticide spraying or introducing sterile flies. This computer simulation package was developed by the Huntington II Computer Project at SUNY at Stonybrook, New York. For more information about the program, you can contact Robert Crifasi at the Essex County Educational Services Commission, Computer Services Division, 91 West Market Street, Newark, NJ 07103 or call (201) 596-1992.



The Mediterranean Fruit Fly, Ceratitis capitata, magnified 16 times.

CAN THE DOUBLING FACTOR BE DEMONSTRATED?

OBJECTIVE: Students should be able to apply the concepts of arithmetic growth and geometric growth to resource depletion problems.

MATERIALS: Peanuts (50 for each student)

PROCEDURE:

1. One can demonstrate the concept of depletion of a resource by giving each student 50 peanuts from a stockpile that represents a finite quantity of the resource. (Note that the number of peanuts supplied each student need not be exact. Count out one set of 50, then estimate other sets by taking comparable volumns.)

2. Then, divide the class in half. To demonstrate arithmetic depletion, have half the students eat one peanut at time zero, two peanuts at 5 minutes elapsed time, three at 10 minutes, four at 15 minutes, etc. To demonstrate geodepletion, the other half of the students should eat one peanut at time zero, two peanuts at five minutes, four at 10 minutes, eight at 15 minutes, etc. These time intervals and peanut counts work well for a 50-minute class period, but can be adjusted for periods of other lengths. Each time interval can represent one year or one century or any other depletion period (for geometric depletion, the time interval represents the "halving time"). Typical counts are given in Tables 1 and 2.

3. A similar demonstration, but one which involves a great deal more work, can be used to illustrate Malthus' law: Food supply grows arithmetically while population grows geometrically; thus, mass declines in quality of human life (and even, starvation) are inevitable unless population is controlled. Supply students with peanuts at an arithmetic growth rate, but have them consume peanuts at a geometric rate. (Start with a slight surplus and with production slightly ahead of consumption.) Typical data are given in Table 3. Many variations of the demonstration are possible. For example, one could have the peanut supply grow arithmetically, as above, and have the consumer population growing geometrically.

Credit: "Sunny Side-Up," Seminole County Schools, Sanford, Florida, Betty Palmer, Science Supervisor.

NAME________________________________DATE__________

TABLE 1. ARITHMETIC DEPLETION

Time (Minutes)	Peanuts eaten at that time	Total peanuts gone
0	1	1
5	2	3
10	3	6
15	4	10
20	5	15
25	6	21
30	7	28
35	8	36
40	9	45
45	10	(55)

TABLE 2. GEOMETRIC DEPLETION

Time (Minutes)	Peanuts eaten at that time	Total peanuts gone
0	1	1
5	2	3
10	4	7
15	8	15
20	16	31
25	32	(63)

TABLE 3. MALTHUS'S LAW

Time (Minutes)	Peanuts supplied	Peanuts eaten	Peanuts remaining
0	(10)	1	9
5	5	2	12
10	6	4	14
15	7	8	13
20	8	16	5
25	9	(32)	-

1. Choose a non-renewable source (oil, coal, gas). Let time be years, the source be barrels, etc. Complete two tables, one using arithmetic depletion and one using geometric depletion.

2. If there are 1,000 barrels of oil left in the earth and 5,000 tons of coal, determine how many minutes it will take to use your resources.

   a. Arithmetic depletion ______________________________.

   b. Geometric depletion ______________________________.

DEMONSTRATING EXPONENTIAL GROWTH

OBJECTIVE: To demonstrate exponential growth

MATERIALS: Graph paper, crayons, stopwatch or clock with second hand.

PROCEDURE: Give each student a sheet of graph paper. Tell them they will fill a unit of the graph in a given time interval. (Thirty seconds is practical.) In the next interval they will double the area shaded, so that one unit is filled, then two, four, eight, and so on. Have a student be time keeper with no pause between doubling times.

If students use a different color crayon for each doubling time, the result will be more dramatic and it will be easier to see the doubled areas. If 30 seconds is used as the doubling time, students will run out of graph paper (resource) in about 4 minutes.

DISCUSSION:

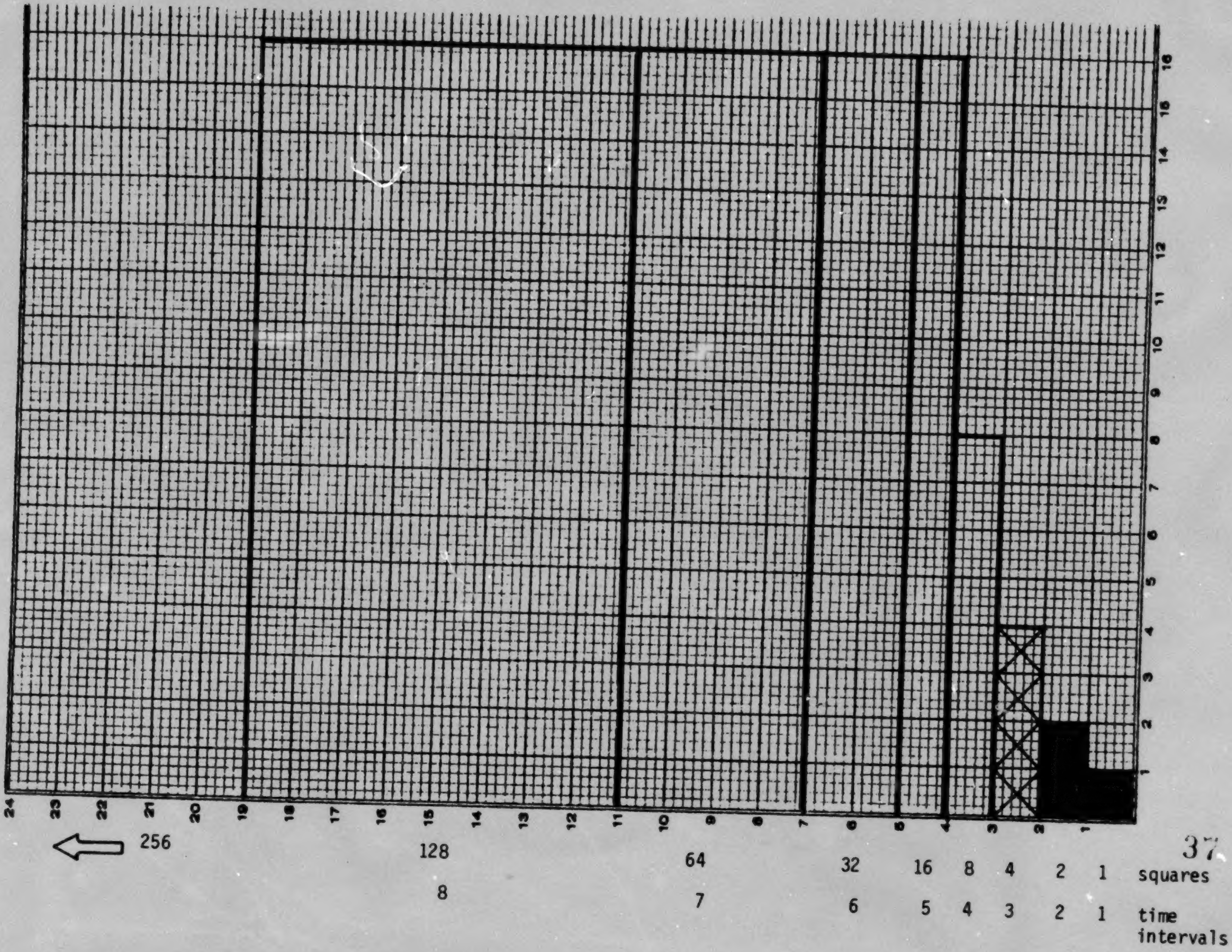
1. If the quantity being doubled is a population of organisms (humans, blue jays, mice, algae, etc.), what will be the effect on their resource needs? Have students list the requirements of the chosen organism (living space, energy/food source, etc.) and discuss the effect of exponential growth on the organism's resources. Will this affect other organisms? How can an organism control it's population growth? How can organisms increase their resource base in the short run, and in the long run?
2. If the quantity being doubled is a resource being depleted, what will be the effect on consumers? How will a resource that is diminishing be allocated?
3. Assign students to find the use rate for petroleum, coal, natural gas, electrical power, etc., in the United States. What are the implications for this rate of consumption? How will this affect wildlife and natural habitats? How will this affect the "quality of life" of the consumers?

DISCUSSION QUESTIONS:

What obligation do we have to provide for future generations? Did past generations pay attention to the needs of the present generation?

How much of a diminishing resource belongs to future generations?

At what rate would use of a resource need to be reduced in order to assure that the resource will last forever?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DEMONSTRATING EXPONENTIAL GROWTH MODULE

DISCUSSION: What obligation do we have to provide for future generations? Did past generations pay attention to the needs of the present generation?

How much of a diminishing resource belongs to future generations?

At what rate would use of a resource need to be reduced in order to assure that the resource will last forever?

PAPER FOLD

OBJECTIVE: Students should be able to:

Use a folded piece of paper to show that linear growth occurs by the addition of a fixed amount, and that exponential growth occurs by doubling the additional amount.

PROCEDURE: Ask students to cut and fold paper for the next activity. This hands-on activity demonstrates the characteristics of both linear and exponential growth.

Distribute student handout.

1. Have students cut a strip 1 inch (2.5 cm) wide from the long side of an 8 1/2" x 11" sheet of paper. Fold the strip every 1 1/2 inch (3 cm) until they have made 6 folds. Cut or tear off the excess beyond the sixth fold. Put the folded strip aside.

2. Tell students to fold the remaining piece of paper in half 6 times. The sheet will become thick and hard to fold.

3. Have students compare the areas of both folded sheets. Point out that both sheets now have approximately the same area. Announce that the area of each represents the year 1920.

Area of the small folded strip	Represents how many loaves of bread can be purchased for an hour of work.
Area of the large folded sheet	Represents the amount of electricity used in the United States.

Have students label each folded sheet.

4. Tell students to unfold the narrow strip. Write the years 1920, 1930, 1940, and so on, in each new unfolded area. After the addition of the last area, the strip represents the purchasing power of an hour of work in the year 1980.

5. Next, begin to show the data for electricity growth between the years 1920-1980. Have students write the year 1920 on the tiny area on top and the words "Use of Electricity--1920" in this small space. For a more dramatic effect at the end, you might suggest students blacken in the area as well.

6. Begin unfolding. Each new area represents electricity use ten years later. In this way it will represent the data for 1930; then 1940; then 1950; and so on. The fully opened sheet represents U.S. electricity use in 1980 (estimated).

7. Conclude the activity by asking:

1. Which one grew more quickly- the number of loaves of bread that can be purchased by one hour's work, or U.S. electricity use?

   (U.S. electricity use.)

2. Which grew by doubling every ten years?

   (U.S. electricity use.)

3. Did electricity use grow at a constant rate? Explain.

   (Yes--It is increasing 100 per cent every ten years.)

4. How would you describe the growth in purchasing power for an hour of work?

   (It is increasing by a constant number.)

Extending the Learning: Discussion Topic

Have students discuss this open-ended question about increased energy use and its relationship to productivity and "quality of life."

Think about why electricity use grew from 1925 to now. Does making more things and making them faster so more people can buy them with money earned from fewer hours of work mean improvements in the standard of living for more Americans? Does it also mean there has been an improvement in the quality of life for most Americans? What is the difference between *standard of living* and *quality of life*?

For students with some knowledge of distances, the power of doubling can be made graphic by citing the following:

If the paper is .003 inches thick, 10 doublings will be over 3" thick. 20 doublings will be over 262 ft., 32 doublings will be over 200 miles (relate to a city that distance from student's home). 34 doublings is about 813 miles (relate to city in mid-west). 36 doublings will take paper past state of Washington. 39 doublings will take paper around the world, and then some (26,030 miles). 42 doublings is 208,240 miles (nearly to moon, 24,000 miles). 43 doublings will go past the moon. 51 doublings will reach the sun. 69 doublings will reach the nearest star, Alpha Centauri (about 4 1/2 light years away). 87 doublings will reach 1.25 million light years (are we still in our galaxy?).

Project for advanced students: find the distance to Alpha Centauri and calculate the number of doublings of paper .003" thick to reach Alpha Centauri. (Fairly easy on pocket calculator.) Student will need to convert from inches to feet, feet to miles, miles to light years when calculator display is nearly full.

NAME____DATE________

Folding and Unfolding

By cutting and folding two pieces of paper, you can compare two kinds of growth.

Cut and fold this sheet as marked. The smaller strip will represent linear growth, as shown by the amount of white bread purchased for an hour's work from 1920-1980. The remaining sheet will represent exponential growth as shown in the amount of electricity used in the United States from 1920-1980.

FOLD THIS SHEET IN HALF 6 TIMES.

CUT OUT AND FOLD ALONG DOTTED LINES.

CUT OFF

EXCESS

NAME____DATE___________

Exponential Growth Plots as a "J" Curve

DIRECTIONS: Using this table, you will plot two increases on a graph. From these lines you will be able to tell how quickly or slowly something is increasing.

The first curve you are going to plot will show the rise in the number of loaves of bread that can be purchased for an hour of work. Look at the first two columns in the table below. Then, plot one line to correspond with the figures in column 2. This curve will show the rise in the amount of bread that can be purchased for one hour of work between the years 1920 and 1980. Then, plot the increase in the use of electricity for 1920-1980, using the figures in the third column. The first decade has been plotted for both curves.

Number of Unfolds	Area of Thin Strip of Paper	Area of Large Piece of Paper
0 (1920)	1	1
1 (1930)	2	2
2 (1940)	3	4
3 (1950)	4	8
4 (1960)	5	16
5 (1970)	6	32
6 (1980)	7	64

Each fold represents 10 years for both papers. Each additional unit of area for the thin strip of paper equals 1 2/3 more loaves of bread that can be purchased for one hour of work.

In 1920, the pay for one hour of work could purchase 5 loaves of bread. Therefore, the first unit of area is equivalent to 5 loaves. Each unit of area for the larger piece of paper represents 60 billion kilowatt-hours of electricity consumed, or the total amount consumed in 1920. But, with each unfolding the area is doubled.

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NAME____ DATE__________

You've demonstrated two kinds of growth: one is linear and one is exponential. The increase in the number of loaves of bread purchased for one hour of work demonstrates linear growth. What is constant is the amount of the increase. The increase in electricity consumption shows exponential growth. Here the constant is the rate of increase of the period of time over which the growth doubles.

Questions

1. Which curve shows exponential growth? Which shows linear growth? What statement can you make about the shape of each curve?

2. What is the amount of increase (every ten years) for each curve? For which curve is the amount of increase constant?

3. What is the doubling time for each curve? For which curve is the doubling time constant?

GROWTH AND OIL

OBJECTIVES: Students should be able to:

Use rectangular coordinate paper to show the area under an exponential curve within one doubling time is equal to all the area under the curve before.

Explain that it is a prerequisite for projecting doubling times that all conditions remain the same.

MATERIALS: Transparency: "U.S. Consumption of Petroleum, 1920-1980."

PROCEDURE: Have two students cover a large bulletin board with butcher paper. Put the histogram (next page) on the overhead projector and focus the image on the bulletin board. Have students trace the histogram in the lower left corner of the wall or door space. Use marking pens for the tracing. The projected size can be made larger than the original, or you may wish to use the same size. However, if you choose to make the tracing larger, be sure to increase the size of the construction paper blocks.

Next, ask students to identify the point on the graph that represents 1980 projected estimate of U.S. oil consumption. (7 billion barrels) ASK: What was American oil consumption in 1978? (6.8 Billion barrels) What do the squares on the histogram represent? (All the oil consumed in the U.S. from 1910 to 1980 estimated.)

Our oil consumption was growing 7% per year. Fortunately, in the last 5 years, the growth of consumption has slowed to about 2% per year.

1. Use the following formula to calculate the present doubling time for U.S. oil consumption.
2. What was the doubling time when consumption was growing at 7% per year?

$$\frac{70}{\text{rate of growth}} = \text{doubling time}$$

$$\frac{70}{2} = 35 \text{ years}$$

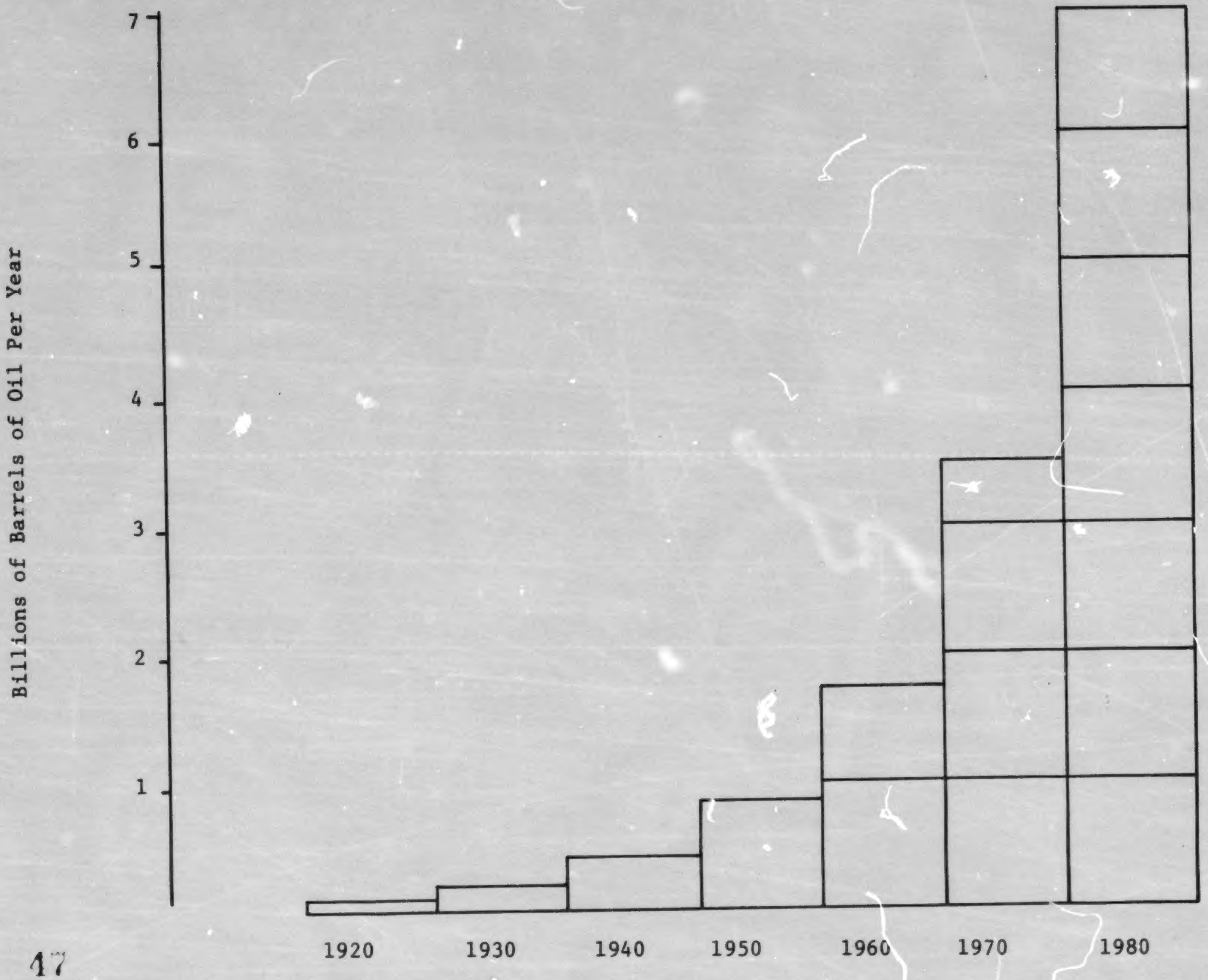
$$\frac{70}{7} = 10 \text{ years}$$

Write the formula on the board. Then, before students do any calculating, stop and ask a student to fill in the areas representing 1920 through 1970 with colored marking pen.

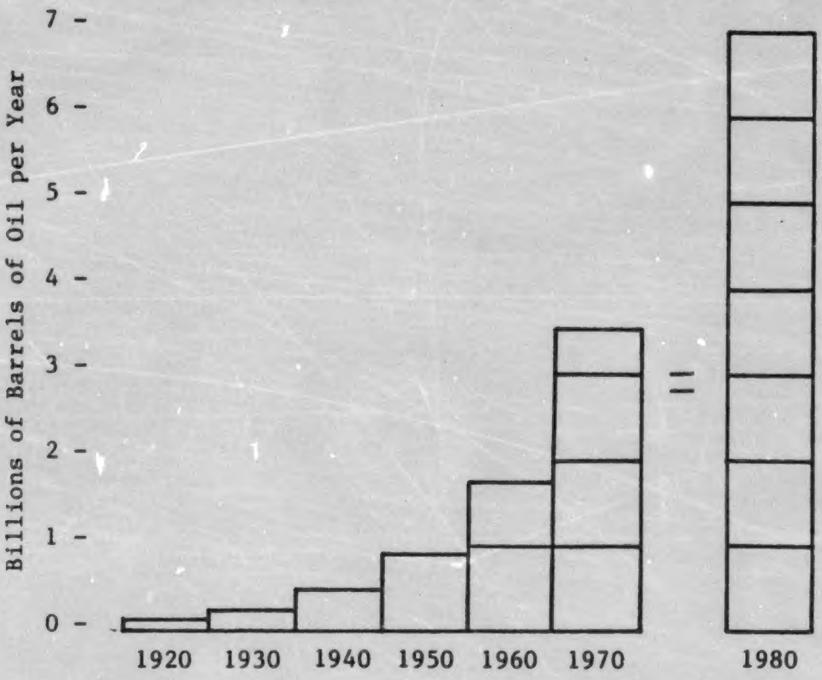
Now, using the formula, ask students how far the horizontal axis needs to be extended to accomodate the doubling time for 7% growth of consumption (10 years). Using the marking pen, have a student extend the horizontal axis on the histogram to the year 1990. Then, have students count the squares representing U.S. oil consumption from 1920 through the end of the 1970s, and count the squares for the 1980s. In other words, the amount of oil we would need in that decade if consumption had continued to grow at 7% would equal all the oil we had ever used. See illustration.

Teacher Transparency Master

U.S. Consumption of Petroleum 1920-1980



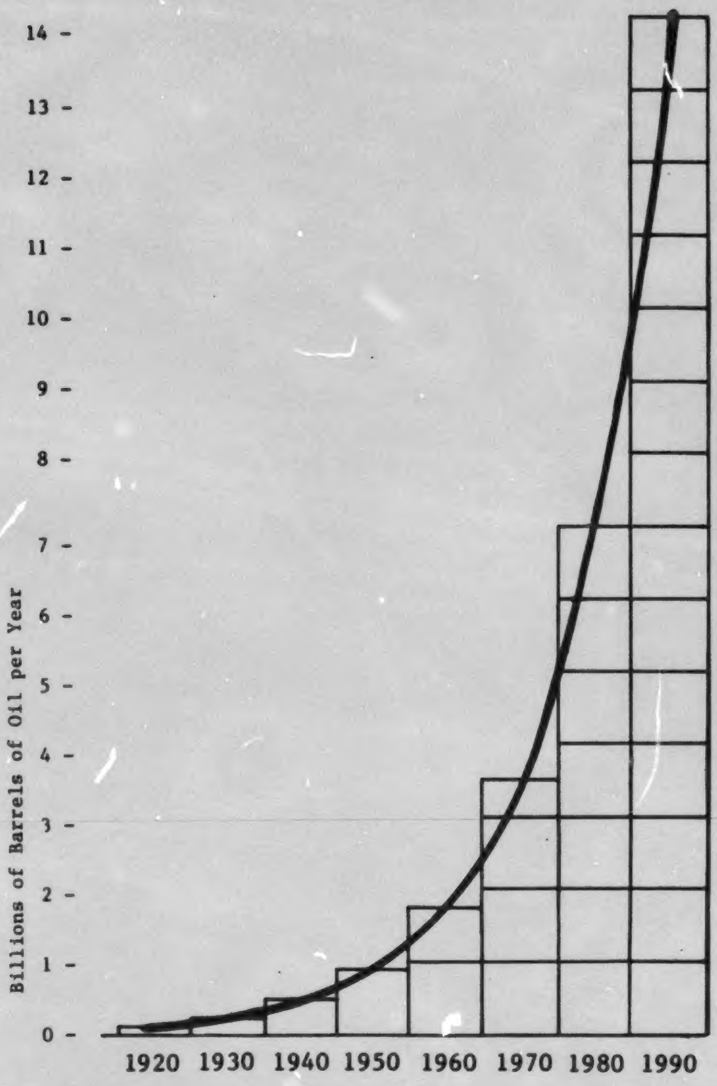
U.S. Consumption of Oil from 1920



Calculate another hypothetical doubling time, using the procedure outlined above. Remember now that all the squares on the histogram have to somehow "fit" into this new 10 year period. Allow students to manipulate the squares once again. Carry out this procedure until the entire class has acquired a "feel" for doubling and exponential growth.

To illustrate an exponential curve, have students plot points in the center of the top square for each decade. Connect the points in a smooth curve. See illustration below:

U.S. Consumption of Oil
at 7% Growth Rate



Write the "bogus" news release in the box on this page on the chalkboard. Ask students if they think this news will end the energy crisis. How long will the new "discovery" fill our oil needs at current growth rates?

(Answers will vary, but students should point out that at 7% growth rate, according to the histogram oil consumed in the year 2000 will equal all we've ever known. Therefore, this "discovery" would accomodate our consumtpion needs for one more decade--for just 10 years.) WHEN THINGS GROW EXPONENTIALLY, ENORMOUS INCREASES IN RESOURCES ARE CONSUMED IN A VERY SHORT TIME.

...UPI...2000 A.D.

Oil has been discovered in Antarctica. This discovery is equal to all the oil consumed by the U.S.--EVER! It appears that the energy crisis is over! U.S. Energy officials are elated. Said one, "We've got oil coming out of our ears!" This is indeed a wonderful day for the people of the United States.

Another good way to evaluate whether students have grasped the concept of exponential growth would be to have them collect advertisements and examine them for misleading statements and tell why these ads tend to mislead. For example: "We have enough coal to generate electricity for 2000 years..." or, "...enough coal to take care of energy needs for many generations..."

(Phrases and statements such as these mislead the reader because the qualifier increase over present consumption is generally omitted.)

NOTES TO TEACHER: If the historical rate of consumption differs from the future rate of consumption, the lifetime is given by

$$X_L = \frac{1}{X_1} \ln \frac{X_1}{X_0} + 1 , \text{ where } X_1 = \frac{\text{new \%}}{100}, \quad X_0 = \frac{\text{old\%}}{100}$$

and X_L = lifetime if the amount remaining is equal to the amount used to date. The key at this level is to make sure $X_1 = X_0$.

DOUBLING TIME

OBJECTIVE: Students should be able to:

Apply the equation for growth, doubling time.

Compute the doubling time of an exponentially increasing demand by dividing the annual growth rate expressed as a percent into 70.

MATERIALS: Dittoed copies of the Handout: "An Equation for Doubling time"

PROCEDURE: In this activity the students compute the doubling time mathematically using an equation. This understanding of the doubling time for various rates of exponential growth should reinforce the students' ability to cope with the often confusing array of numbers associated with increasing growth--consumption, depletion, and projected demand.

Natural increases or decreases, where the change is a fixed percentage of the existing amount, proceed logarithmically with base 3 = 2.71828 (a logarithm base discovered by John Napier). The growth curve (increasing amounts) is expressed by the equation:

$$a(t) = a_0 e^{Rt}$$

$a(t)$ is the amount changing with time.

a_0 is the amount that is already there.

R is the fixed annual growth rate expressed as a decimal.

t is the amount of time in years.

The doubling time is when $a(t) = 2a_0$. Substituting this into the above equation and solving for $t = T_2$:

$$2a_0 = a_0 e^{RT_2}$$

$$2 = e^{RT_2}$$

$$\ln (2) = RT_2 (\ln e)$$

natural log of 2 = $.693 \simeq .70$

$$\therefore RT_2 \simeq .70 \text{ or } R \text{ (as\%)} T_2 = 70$$

Answers to questions:

The graph represented a doubling of the use of electricity every 10 years. Calculate the growth rate (%) per year:

$T_2 = 10$ years $\qquad R = 70/T_2$

$R = 7\%$ per year

Calculate the time necessary for these exponentially increasing quantities to double:

1978	Quantity	Growth Rate (R)	Doubling Time (T_2)
World Population	4.2 billion	1.7%/yr	41 yr.
Money in savings	$100	5% interest/yr	14 yr.
Cost of loaf of bread	$0.50	6% inflation/yr	11 2/3 yr.
US oil production	3.7 billion barrels	3.25% increase/yr	21 1/2 yr.
US oil demand	6.8 billion barrels	2.3% increase/yr	29 1/2 yr.

NAME____ DATE_______________

An Equation for Doubling Time

An exponentially increasing quantity doubles at a constant rate. For instance, a quantity may double every 12 years. Another quantity increasing at a faster exponential rate might double every 2 years.

An exponentially increasing quantity grows at a fixed percent each year. The doubling time in years can be computed by dividing the growth rate per year expressed as a percent into the number 70:

$$T_2 = \frac{70}{R}$$

R = fixed rate of growth in percent per year

T_2 = doubling time

Example: Given growth rate \$ = 2% per year
Doubling time T_2 = 70/2% = 35 years

Example: Given doubling time T_2 = 70 years
Growth rate \$ = 70/70 years = 1% per year

NOTE: When $R > 15\%$ the formula loses accuracy and should be replaced by $T_2 = \dfrac{70}{\ln\left(1+\frac{R}{100}\right)}$

For $R < 15$, $\ln\left(1+\frac{R}{100}\right) \simeq \frac{R}{100}$

R/100	$\ln\left(1+\frac{R}{100}\right)$
.05	.048790
.10	.095310
.15	.139762
.20	.182322
.30	.262364

Student Handout

NAME____DATE__________

Now, calculate the time necessary for these other exponentially increasing quantities to double:

1978	Quantity	Growth Rate (R)	Doubling Time In Years
World population	4.2 billion	1.9%/yr	________
Money in Savings	$100	5% interest/yr	________
Cost of loaf of bread	$0.50	6% inflation/yr	________
US oil production	3.7 billion barrels	3.25% increase/yr	________
US oil demand	6.8 billion barrels	8.25% increase/yr	________

USING A POCKET CALCULATOR TO DEMONSTRATE EXPONENTIAL GROWTH

PROCEDURE: Use a calculator that has a % key and have students compound a number at various rates 7%, 10%, 16%, etc. Have them note the number of increments before the original figure doubles.

Introduce the formula for doubling time, $T_2 = 70/P$ where P is the percent growth per unit time and T_2 is the doubling time. Recalculate using the formula and the same percentage rates as before.

Relate these concepts to everyday life, i.e., compound interest on savings accounts, population growth worldwide, the use of coal in the United States, the rate of burn of rocket fuel in a space launch, the amount of land converted to housing annually in the U.S. and so forth.

The calculator can be used as a convenient tool to rapidly explore the impact of exponential growth.

Pocket Calculator Module

A programmable calculator provides a very nice way to demonstrate exponential growth. All that is needed to program the calculator to calculate

$N = N_0 (1 + X)^n$, where

N = amount after n years (seconds, days, etc.)

N_0 = initial amount

X = percent growth rate expressed as a fraction (i.e., 5% = .05)

The program on an H-P 25 would look like this:

Initialize: Store (1 + X) in register 0.
i.e., (1 + X) STO 0.

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```
            Step        Function

Program:    01          RCL 0
            02          (X)
            03          Pause )
            04          Pause { These pauses give time to
            05          Pause ) see result
            06          GTO 1

Execution:  Enter N0
            Run
```

The program can be made as fancy as the calculator can handle. The next step might be to have the calculator display the number of time increments, n, followed by the display of N. (See next page for program.)

Next, try including accumulation, so that T(n) = N(1) + N(2) + N(3) + --- + N(n) is stored and displayed.

Finally, compare T(n) with the estimated reserves and have the calculator stop when the resource runs out.

Microcomputers are nice for plotting results.

Program to display n and N(n) on H-P 25.

```
              (1 + x)      STO 0
                   No      STO 1
Initialize         1       STO 2
                   0       STO 3


              Step         Function

              01           RCL 3
              02           RCL 2
              03           (+)
              04           Pause }
              05           Pause }     Displays n
              06           STO 3
              07           RCL 0
              08           RCL 1
              09           (x)
              10           Pause }
              11           Pause }     Displays N(n)
              12           STO 1
              13           GTO o1
```

Program to display n, N(n) and T(n) on H-P 25

(1 + x) STO 0
N_o STO 1
Initialize N_o STO 4
1 STO 2
0 STO 3

```
Step          Function

01            RCL 3
02            RCL 2
03            (+)
04            Pause ⎫
05            Pause ⎭     Displays n
06            STO 3
07            RCL 0
08            RCL 1
09            (x)
10            Pause ⎫
11            Pause ⎭     Displays N(n)
12            STO 1
13            RCL 4
14            (+)
15            Pause ⎫
16            Pause ⎭     Displays T(n)
17            STO 4
18            GTO ol      Loop Command
```

PART THREE...............

ANOTHER WAY TO VIEW EXPONENTIAL GROWTH

The lessons in this section offer an alternative way to get at an understanding of exponential growth. That way is a speculative, personalized, creative one for students to experience exponential growth as an idea and as a reality.

The following activities are designed to help students develop insights into the patterns of arithmetic and geometric (or exponential) growth.

Most of the activities are relatively ambiguous in their assignment format in order to stimulate students to utilize their personal life experiences to connect to the exponential concepts. The activities are also designed to encourage auditory, visual and kinesthetic modalities for expressing student insights.

Experience has indicated that if students can accomplish sharing their insights to concepts at these levels and gain insight to conceptual patterns, intellectual growth is facilitated.

The result of ambiguity in the assignments should, when they are shared, provide student with divergent insight to a wide range of conceptual understanding--no single type response is likely to emerge when the class is given an ambiguous assignment..many will, and students will make the connection of these abstract ideas to their own personal lives.

We hope teachers will use a diversity of activities from this guide for it is a balanced delivery of ambiguity and specificity that all learners are shared.

Experience, too, has indicated ambiguous activities are excellent as introductory activities as they act as a conceptual status assessment of student insight for teachers, while building relevancy and diverse conceptual insight for students. They make good change of pace activities, homework or project assignments or culminating or wrap up summaries.

Exponential Growth Patterns

1. Cut out 5 pictures from magazines and put them in (1) a sequence that demonstrates exponential or geometric growth and (2) another series that demonstrates arithmetic or straight line growth.

2. Line up the class in single file. Now form a line that would look like the growth graph

   - of a rapidly growing city with twice as many new people every ten years

   - a community growing 10 additional people each year

   - a viral infection growing in your body

   - an algae bloom in a lake

   - traffic on a highway at 4:00 a.m.; at 5:15 p.m.

3. Bring in two examples that show how you are part of a geometric growth pattern and also part of an arithmetic growth pattern.

4. Collect a series of sounds that illustrate a geometric growth curve in progress.

   Collect a series of sounds that illustrate an arithmetic growth pattern. (Is this in terms of decibels?)

5. Create a 3-D model of a system growing arithmetically.
   Create a 3-D model of a system growing geometrically.

6. Find an example of something in the community that has grown both arithmetically and geometrically.

7. Find an example of how arithmetic growth is affecting your life positively and another example of how arithmetic growth is having a negative effect on you...repeat for geometric growth.

8. On a piece of 8 1/2 x 11 ditto paper place one piece of 1 cm square colored paper. Then, place two squares on the paper and repeat the process, each time doubling the number of squares on the sheet.

   - how many doublings did it take to half cover the page with squares?

   - how many doublings did it take to completely cover the paper?

   - how fast can you cover the paper with this procedure? Who in the class had the fastest rate of doubling; who had the slowest? Create a math picture graph that illustrates both rates slow and fast.

9. Each team of two students:

   Gather six gambusia (mosquito fish) from a pond or ditch or purchase six Guppy tropical fish (3 - 3). Design a study (an ideal environment) that will maximize or encourage the largest possible population of fish in a one month period. Keep a record of how many fish are born and how many survive to reproduce - how many die? Create a graph that shows the pattern of population growth of fish! Compare graphs and show the reasons they are similar or different.

10. Collect from music lyrics you know those that show arithmetic growth and those that show geometric growth and put them together in a way that creates a new growth song.

11. Invent a way of showing how arithmetic and geometric growth are alike...are different.

12. Give examples of how ideas or feelings grow arithmetically and geometrically.

13. Find an example of two geometric growth systems that are working against each other....Find two systems geometrically growing in harmony.

14. Find an example of something growing arithmetically that is larger in quantity than another thing that is related but growing geometrically.

15. Pretend you are the leader of a group of microcreatures that must shift from growing in population arithmetically to a geometric growth pattern. Write a set of rules that will guide the groups to shift to geometric growth patterns.

    Repeat this activity for creatures shifting from geometric to arithmetic patterns. How are your rules for the two groups similar...How are they different?

16. Create a skit that demonstrates imaginary hypothetical creatures experiencing exponential growth...arithmetic growth... no growth.

17. Create a T.V. commercial that sells the advantages of arithmetic growth over geometric growth or vice versa.

18. Move like or create an invention that will move like:

    A growth system in equilibrium

    A high rate arithmetic growth pattern

    A low rate geometric progression pattern

    A negative geometric progression pattern

    A high rate geometric progression pattern

    A system which switches growth patterns

19. Create a work of art which illustrates geometric growth or arithmetic growth patterns or both patterns of growth.

20. Create a way of teaching another person the difference between arithmetic growth and geometric growth...then try it out!

21. Create a class newsprint mural which illustrates all the class knows about arithmetic and geometric growth patterns rules, forms and examples in life.

22. Create a mobile with thread and objects you collect that illustrates geometric and arithmetic growth at positive and negative rates.

23. Make a collection of different textured objects that you associate with arithmetic growth and with geometric growth patterns.

24. Collect and display a color or hues that you relate or associate with arithmetic growth and a color or hues you associate with geometric growth and color or hues you associate with no growth.

25. Find examples of growth you like and examples of growth you don't like.....Are your "likes" mostly growing in arithmetic or geometric patterns?....What do you think makes you feel the way you do?

26. Complete the following story.....It is 1983 and you are living in Florida sharing the sunlight, air, water, soil, wildlife while the Earth's population has reached 5 billion people. Suddenly you are transformed into the year 2025 and there are 10 billion people on the earth sharing those same limited resources with you and you now find your life is like.... (complete the story)

/Option/ Write a story telling your grandchildren living in the year 2025 what life back in 1980 in Florida was like.

27. Take your class outdoors in a comfortable, quiet place and have them create a story by allowing one person to start the story on the theme of Exponential Growth. Each successive student can add up to four sentences to the story. The focus of the story should take place on the school grounds. If possible, tape record the story for play back discussion later. Allow students' imagination to flow freely.

28. Ask students to write three operational and three non-operational definitions of exponential growth. Students should share definitions with the class and the class decide which category the definitions fit. A non-operational definition tells what something is or of what it is made. An operational definition tells how to measure something or how you can tell if you have a certain amount of something.

29. Find a population in nature which is growing...Determine if it is growing arithmetically or geometrically. Share the evidence you used to reach your conclusion in writing or orally.

30. Interview three people and find out what rate of growth % they would like to see in your community. Find out the present population of your city or town. Calculate at the % rates people gave you how long it will take your city to double its size (see appendix for formula).

31. Automobiles and Trees

Emphasize with your class how green plants are the only source of oxygen on the earth. For every automobile that emits carbon dioxide into the air, fifty large healthy trees are needed to restore the oxygen that was used. The class should study the school grounds, working in pairs or small groups and count the number of cars in the parking lot versus the number of trees on the school grounds (small trees or shrubs should be counted only as fractions of a "large healthy tree"). Do the school grounds have an oxygen surplus or deficiency? How many more trees would be needed to create and oxygen balance? Discuss the added effect of cars driving past the school. Have students suggest from a map which local areas should have an oxygen surplus to compensate for the reduction in their area. What would happen to school gound oxygen levels if the number of cars in the school neighborhood doubled each month for the next three months?

RECOMMENDED FILMS

* Tragedy of the Commons
* Cosmic Zoom
* Sorcerer's Apprentice
* Energy to Burn
* Toast
* Population Ecology

PART FOUR.......................

THE CONSEQUENCES OF EXPONENTIAL GROWTH

The lessons in this final section provide the opportunity to reflect upon the consequences of exponential growth in the realms of energy and environmental consumption.

EXPONENTIAL GROWTH IN ORANGES

OBJECTIVE: To inform students of the doubling method and its importance to them and future generations, by using familiar examples.

MATERIALS: Pencils/Paper

PROCEDURE: Begin with a single orange tree. Explain to the students that a single healthy tree produces approximately 1000 oranges per year.

Propose the questions, that if we took an average class size of 30 students and each student ate a single orange the first day, and each day following we doubled the number of students eating an orange, how long then would our 1000 oranges last?

EXERCISE:

No. of Days	1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day
No. of Students	30	60	120	240	480	960
Single Tree Producing 1000 oranges	30	90	210	450	930	1890

As the students can readily see by the end of the 5th day, we wouldn't have enough oranges to feed the population on the sixth. Propose then the following questions to generate discussion.

POSSIBLE QUESTIONS:

1. How many days would our food supply of oranges last? (5 days)
2. If oranges were one of our main sources of food, what would happen on the sixth day? (Answers vary)
3. What would happen to the price of oranges? (Rise in cost)

4. What can be done to insure having enough oranges from the sixth day forward? (Answers vary)

5. If, on an average acre of land, 100 trees can be planted, each tree produced 1000 oranges per year and the total acreage available for the planting of orange trees in Florida is 8,389,743, how long, then, would the oranges last by using the doubling method?

6. If, on an average acre of land 100 trees can be planted, each tree produced 1000 oranges, then by using the doubling method, how long would the hundred thousand oranges last us?

Question #5: Total Number of oranges = 838,974,300,000

100,000 oranges per acre x total acreage available 8,389,743

$$\begin{array}{r} 8{,}389{,}743 \\ \underline{\times\ \ 100{,}000} \end{array}$$

838,974,300,000 oranges

Question #6: Total number of oranges = 100,000

100 trees x 1,000 oranges per tree

$$\begin{array}{r} 1{,}000 \\ \underline{\times\ \ 100} \end{array}$$

100,000 oranges

CHART TO DEMONSTRATE QUESTIONS 6 and 7

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
NO. OF STUDENTS	30	60	120	240	480	960	1920																				
8,389,743 available acres producing 838,974,300,000 oranges per year	30	90	210	450	930	1,890	3,810																				

	28	29	30	31	32	33	34	35	36	37	38	39	40
NO. OF STUDENTS	3,982,352,640	7,964,705,280	15,929,410,560	31,858,821,120	63,717,642,240	127,435,284,480	254,870,568,960	509,741,137,920					
NO. OF FRUIT PER YEAR 838,974,300,000	7,864,705,150	15,729,410,300	31,458,820,600	62,917,641,200	125,835,282,400	251,670,564,800	503,341,129,600	1,006,682,259,200					

Question 6: Would not have enough oranges to feed the population on the 35th day.

Question 7: Would not have enough oranges to feed the population on the 12th day.

* Use and Cumulative (practically the same one day apart)

Day	# Student	Cum Total
7	1,920	3,810
8	3,840	7,650
9	7,680	15,330
10	15,360	30,690
11	30,720	61,410
12	61,440	122,850
13	122,880	245,730
14	245,760	491,490
15	491,520	983,010
16	983,040	1,966,050
17	1,966,080	3,932,130
18	3,932,160	7,864,290
19	7,864,320	15,728,610
20	15,728,640	31,457,250
21	31,457,280	62,914,530
22	62,914,560	125,829,090
23	125,829,120	251,658,210
24	251,658,240	503,316,450
25	503,316,480	1,006,632,930
26	1,006,632,960	2,013,265,890
27	2,013,265,920	4,026,531,810
28	4,026,531,840	8,053,063,650
29	8,053,063,680	16,106,127,000 *
30	16,106,127,000 *	32,212,255,000
31	32,212,255,000	64,424,509,000
32	64,424,509,000	128,849,019,000
33	128,849,019,000	257,698,038,000
34	257,698,038,000	515,396,076,000
35	515,396,076,000	1,030,792,152,000

This example illustrates the principle of exponential accumulation very nicely.

PRINCIPLE: The amount consumed in a doubling time approximately equals the amount consumed in all previous history. This approximation gets better as n gets larger.

Observe entries in Chart A. Compare day n # of students with total number of oranges eaten by the end of day (n-1). Note that difference remains constant at 30.

# Students on Day n	Total # of Oranges eaten by end of day (n-1)
60	30
480	450
3840	3810
7,864,320	7,864,290

Additional Examples

1. There is 438,791 acres available in Florida for the raising of beef cattle. Each acre of land can support approximately 5 cows. The total number of beef cattle in Florida is 2,193,955.

QUESTION: If the average student ate 1½ lbs. of beef a day, and the the average cow dressed out to 550 lbs, using the doubling method, how many days will it take to consume the 550 lbs. of beef?

No. of Days	1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day	7th Day	8th Day	9th Day	10th Day
Single Student	1	2	4	6	8	16	32	64	128	256
1 Cow 550 lbs.	1½	4½	10½	19½	31½	55½	103½	199½	391½	775½

As you students can readily see on the 10th day there wouldn't be enough beef left to feed the population.

This activity can be used with any number of students and any number of cows.

2. This method can be used to show almost anything you wish to convey to your students. In order for them to understand the energy and environmental crises, they must understand exponential growth. The following activity will demonstrate population growth.

BACKGROUND: Probably the most significant cause of the energy/environmental predicament is the phenomenon ot exponential growth. Over the past few decades, population, non-renewable resource consumption, food production, industrial output and pollution generation have all been increasing exponentially. To understand the energy and environmental crises, one must understand exponential growth. This activity is designed to give that understanding. It is built around the population problem, but can be applied to automobile production and disposal, electrical generating plant construction and demolition, etc.

PURPOSES:

1. To use dice to model exponential growth.
2. To change the variables and attempt to slow down and half exponential growth.

PROCEDURE: In this activity you will use dice or colored wooden cubes to model population growth. Each die or cube represents a person. Each throw represents a year. The directions are written assuming you are using dice. If you use wooden cubes, make the substitutions shown below:

event	dice	cubes
birth	3 or 6	white,red
death	1	black

A "3" or a "6" represents the birth of a child, so each time one of them comes up, add a die to the population. If a "1" comes up, a death has occurred, so remove it from the population. Hence you are modeling a situation where the birth rate is twice the death rate. You also have a population growth rate of 1/6 or about 17%.

2a.

SAMPLE CONCLUSIONS:

1. Population vs. Time

   When population grows exponentially and unchecked, it grows slowly when the population is small and then rapidly increases once a significant population base is established.

   The limited birth control program slows the overall growth; but since it is still exponential, it can get out of hand if given enough time.

2. Growth Rate vs. Time

   As the population increases, so does the growth rate. It appears that the growth rate also increases exponentially with time.

3. Growth Rate vs. Population

   Averaging out the randomness which is very pronounced when one deals with small numbers of objects, one can see that the best graph is a straight line which passes through the origin.

   HENCE: Growth Rate vs. Population

4. Exponential growth is indicated by a straight line on semi-log paper. This can serve as a test for exponential growth.

   The limited birth control program reduces growth, but the growth is still exponential.

   The ZPG plan stops exponential growth.

5. World Population vs. Time (1650 - 1976)

   This graph looks very similar to the one obtained for unrestricted exponential growth. World population increases slowly from 1650 to 1900, when technology and medicine were relatively undeveloped. However, after 1900, the curve shoots up rapidly. Now, world population increases more in a few decades than it did in a few centuries before 1900.

DISCUSSION: Discuss the graphs and the conclusions for each one. What you do from there depends upon your class, your priorities and the attitudes prevalent in your area. It is easy to say that exponential growth can be stopped by equating the birth rate to the death rate. How this is actually accomplished is another story altogether.

MATERIALS: You will need three-cycle semi-log paper. This can either be purchased or crudely drawn out and duplicated. (Use the K scale of a slide rule for making out the log scale and a ruler for making out the linear scale.)

The activity is best accomplished in groups of 6 to 8 students gathered around a table. Dividing up the labor saves a great deal of time and is a great exercise in group cooperation.

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YOUR ANCESTORS

MATERIALS: No materials are required. A colonial period map of the world could be used to assist in demonstrating some points.

METHOD: A general discussion of ancestors, famous persons in history, and some historical events.

- For ease of computation let us say the students were born in 1975. At that time they had two parents. Using age 25 as the approximate age at which each becomes a parent, there would have been four grandparents involved at the time of the birth of their parents.

- Using exponential growth let each student compute the number of ancestors to be added each 25 years (going backward in time):

8, 16, 32, 64, 128, 512, 1024, 2048, etc.

- Pick points of interest in world history, for example, at the time of the Revolutionary War (1775) which would be about nine generations. Each student would have had 512 ancestors.

Jamestown was founded in 1607 at which time the student's 16th generation ancestors would have been living, a total of 65,536.

When Ponce de Leon came to Florida in 1513 the 19th generation of parents would be living and there would be a total of 524,288 ancestors and when Columbus came to America there would have been more than a million ancestors in existence.

- Each student with any Anglo-Saxon blood in their veins can claim they had an ancestor with William the Conqueror when he came to the British Isles in 1066. Point out that, duplicate blood lines excepted, each student would have had more than 137 billion ancestors somewhere on this earth at that time. The total population of the earth was much less than that sum in 1066.

- Point out other comparisions for other ethnic groups. African natives were first brought to America as slaves in 1619 and each student would have had about 32,768 ancestors at that time. If you are Italian there were 11,600,000 inhabitants in 1500 of which 1,048,576 were yours.

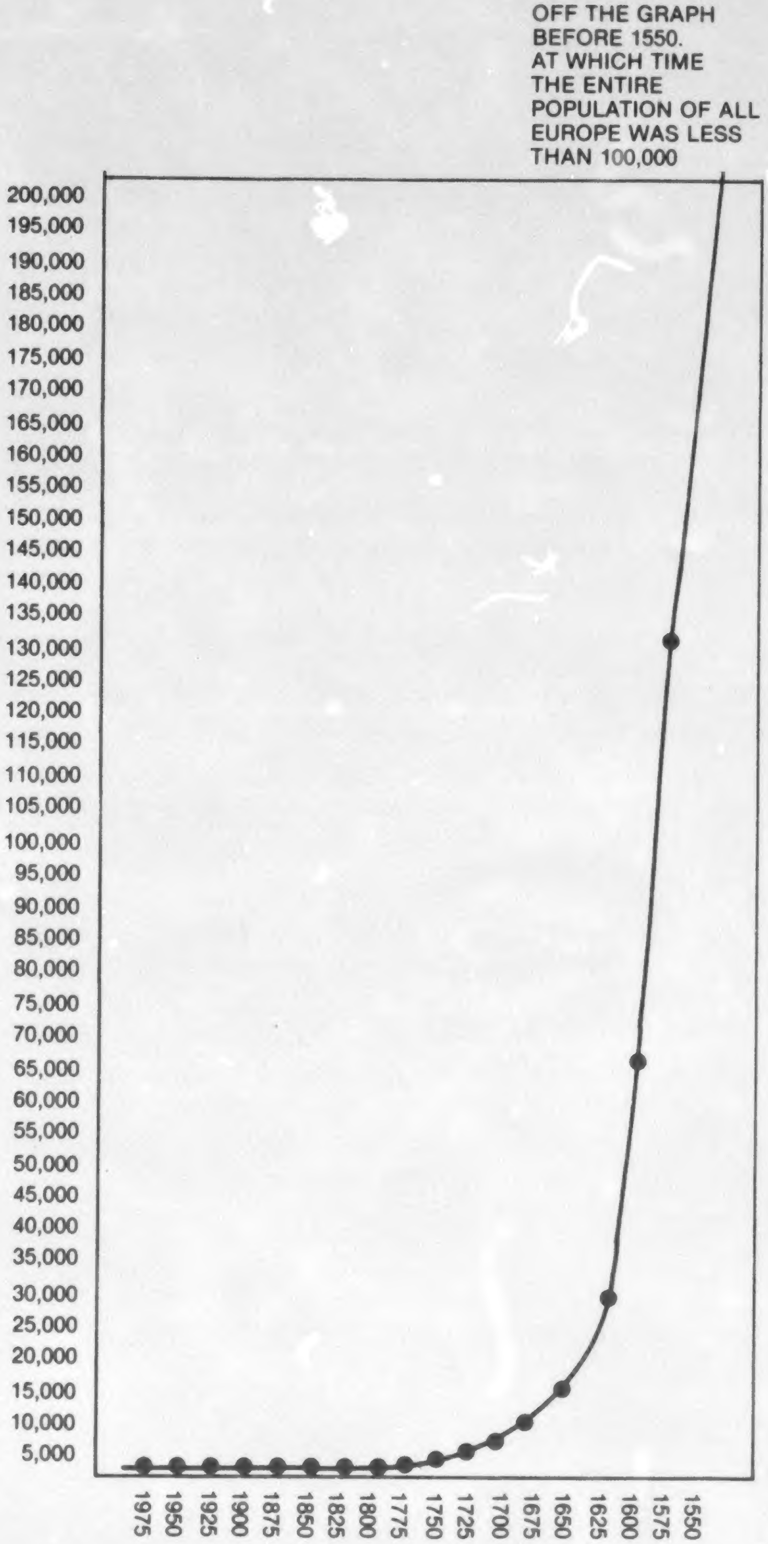
Students should construct a line graph. This demonstrates the phenomenal increase after a few generations.

How many ancestors of yours fought in the Revolutionary War?

Where were your ancestors when Columbus discovered America?

Did you have an ancestor on the Mayflower?

YOUR ANCESTORS



YOUR DESCENDANTS

MATERIALS: No materials are required. Some pictures of the conditions under which children live in some third world countries would be effective to emphasize the situation as it could be with extreme overpopulation.

METHOD: Discuss what is meant by exponential growth. Discuss how the increase appears insignificant in the early stages and how it increases by "leaps and bounds" as the exponent becomes larger.

As practice with exponential growth computations let the student consider what the effect would be if each of the 30 students became the parent of two children and each generation did the same. Then compare that with the results of each student becoming the parent of five children and their children did the same at the rate of a new generation each 25 years and each lived for an average of 75 years.

It would look something like the following:

30 Students		Years	30 Students	
2 children each	Living		5 children each	Living
60	90	25	150	180
120	210	50	750	930
240	420	75	3750	4650
480	840	100	18,750	23,250
960	1680	125	93,750	116,250
1920	3360	150	468,750	581,250
3840	6720	175	2,343,750	2,906,250
7680	13,440	200	11,718,750	14,531,250

QUESTIONS: Under what conditions would your great grandchildren be living if Florida's population increased at the rate of 5 children each generation? (Here pictures would be effective.)

Students would devise a method of demonstrating this growth, for example. See attached graph.

NOTE: It should be noted that these tables are for single parent families. If we allow the 30 to marry each other and become 15 married couples having 2 children each, the picture looks different. See the following table.

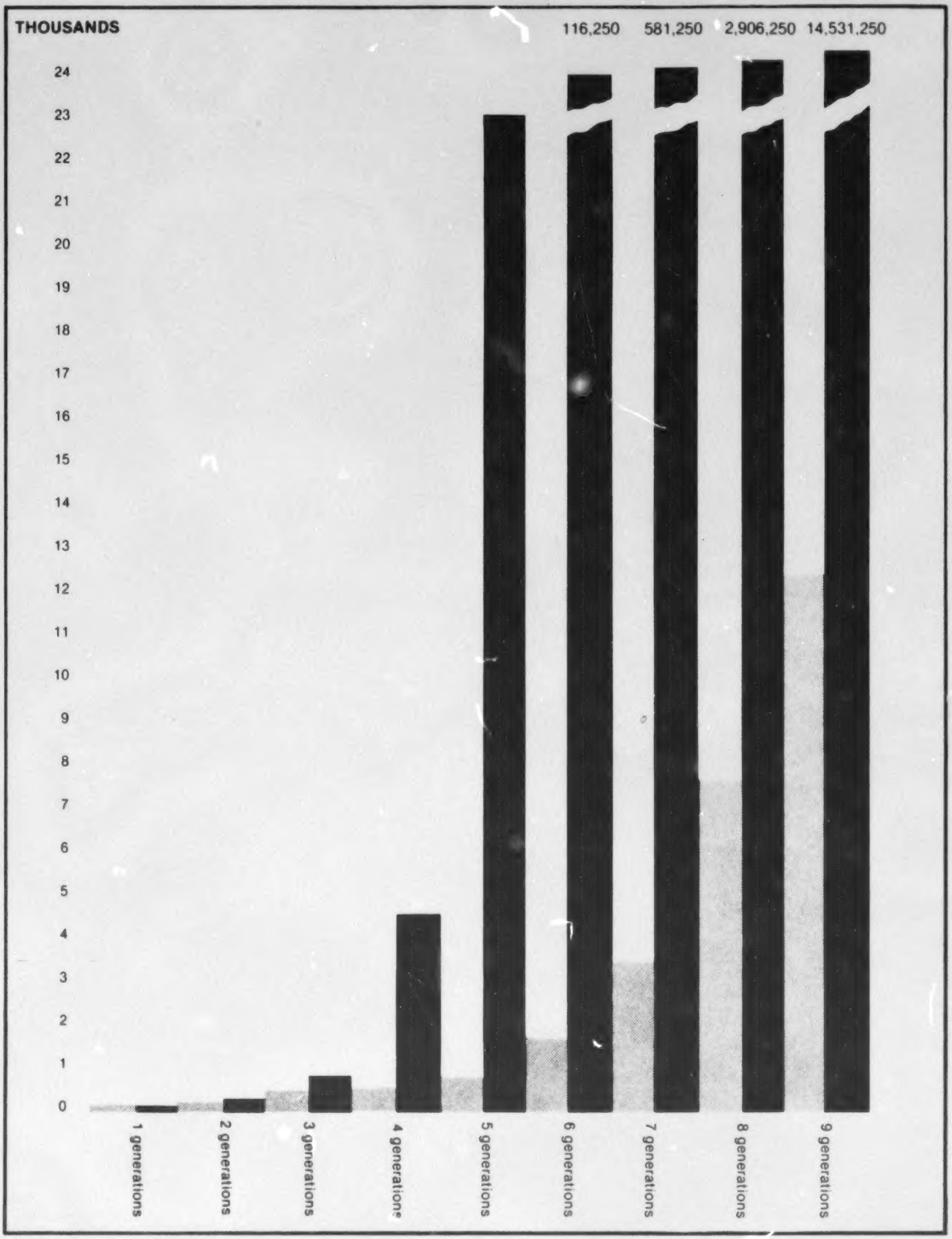
30 Students - 15 couples (if 15 M and 15 F)

	2 Children per couple	Living	Years
	30	30	0
Children	30 15 new couples	60	25
Grand-children	30	90	50
	30	90	75
	30	90	100
	30	90	125

So population alive levels at 90.

NOTE: The 2 children per student is really 4 children per family and 5 children per student is 10 children per family.

GRAPH—WITH YOUR DESCENDANTS



LAND USE IN FLORIDA

OBJECTIVE: Make the student aware that the peninsula on which we live is limited in space.

MATERIALS: A map of Florida and a copy of the Florida Atlas

METHOD: Discuss the present use of land in Florida. Point out the fact that there are approximately 38,000,000 acres in Florida.

Have students determine the present use of land in Florida from the atlas or other resource publications.

Discuss the fact that:

8,389,743 acres are used for agriculture
11,443,000 acres are wetlands and waterways
15,000,000 acres are forest land and other open land
1,000,000 acres are beaches and barren land, and
2,000,000 acres are in urban developments.

With approximately 2½ million acres used for 10,000,000 citizens to live on and to provide highways and business services for the citizens, the student can compute the fact that about 1 acre is needed for each 4 residents.

The present growth in Florida is approximately 50% each decade. If the present rate of land use and population growth continue, the student can determine that by:

1990 - 3 3/4 million acres in urban areas for 15 million
2000 - 5.6 million acres for 22.5 million

QUESTION: Where do we get the 3 million acres needed for the use of the ciizens in the next 20 years.

Have any of you seen a citrus grove removed to make way for a housing development or a shopping center?

What other type of land has been converted to housing?

During the lifetime of the present middle school student, say through 2010 and 2020, the suggested growth will be:

2010 - 8.1 million acres for 33 3/4 million people
2020 - 12.+ million for 50.+ million people

Then to the student:

Where will your children live in 2030 and 2040 if:

2030 - 18+ million acres for 75 million people
2040 - 28+ million acres for 113.8 million people

The only land not in use for residential services would possibly be the 11 million wetland and waterways. No food production, no recreation areas, so no tourists to bring money.

FLORIDA'S POPULATION

OBJECTIVE: To examine past, current and future trends in Florida's population growth.

MATERIALS:
- Chart 1
- Graph paper
- Butcher paper and markers
- String

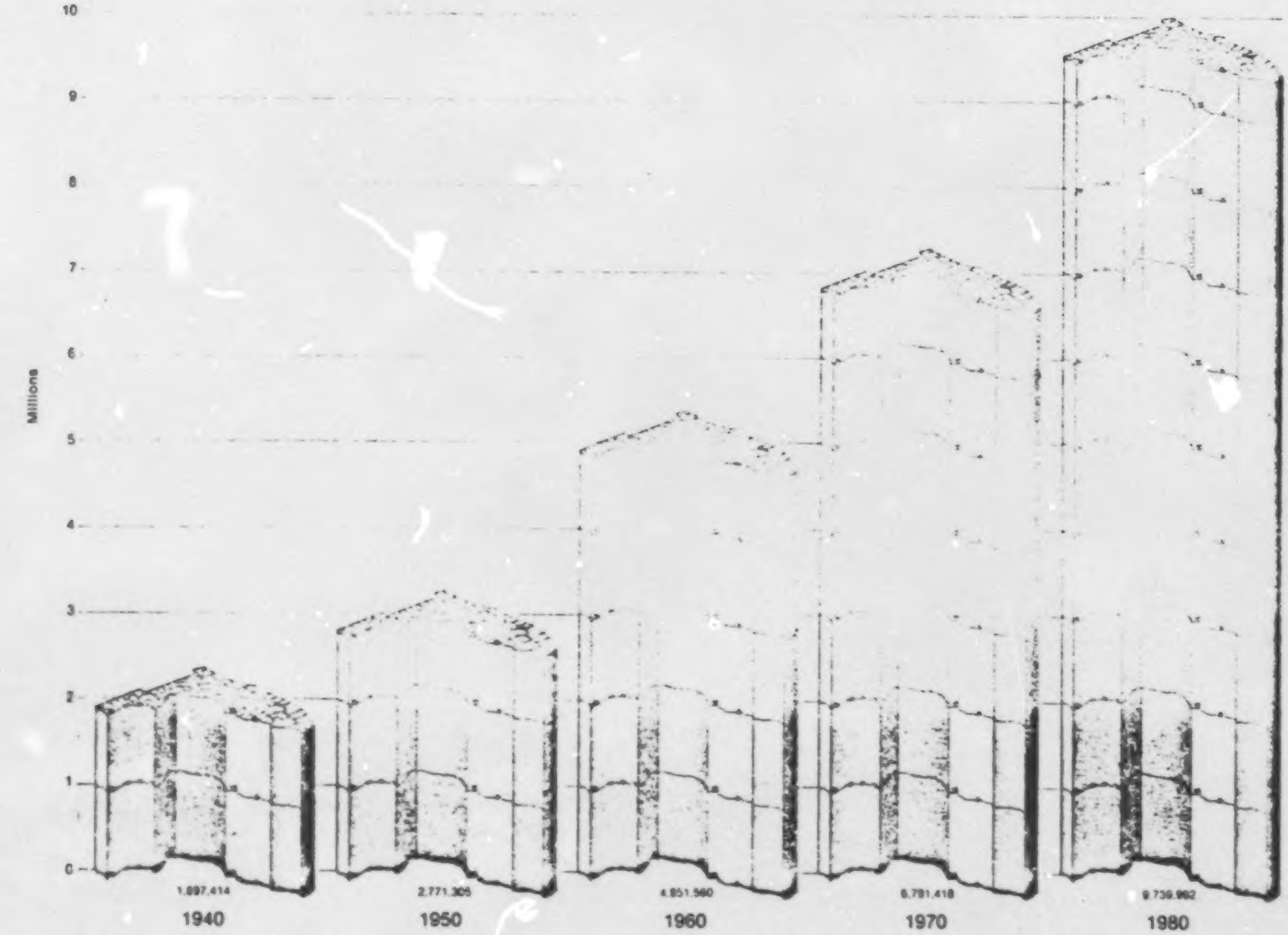
BACKGROUND INFORMATION:

Florida's population growth since 1940 has been phenomenal, and there is no indication this trend will abate. Throughout the last century the state's major attraction was economic opportunity. Despite severe environmental obstacles to the development of the state's resources, technology eventually made their exploitation profitable. As a result more people came to Florida to seek their fortunes in agriculture, forestry, mining and other ventures. Florida's population, however, did not exceed one million until the 1930s. During the depression of the 1930s the state's growth rate fell to an all-time low, but after World War II it began to rise sharply. The federal government, attracted by the mild climate and large amount of public land, built many military training facilities in Florida. During World War II hundreds of thousands of servicemen spent time in the state, and many of them returned to Florida to live after the war.

In the 1960s the Kennedy Space Center served as a stimulant for industrial expansion, bringing more people into the state. The 1959 Cuban revolution had a profound effect upon the state's population. Most of the hundreds of thousands of Cuban exiles who arrived in the United States chose to live as close to their native country as possible. By 1981 Miami had the largest Cuban population in the nation. Economic and political problems persist throughout Latin America, and Florida is a principal destination of Latin American immigrants.

CHART 1

Population Growth



Atlas of Florida, Ed Fernold

1. Round the numbers in Chart 1 to the nearest 100,000 and complete Table 1.

TABLE 1

Population Growth in
Florida 1940 - 2000

YEAR	POPULATION X 10	% INCREASE
1940	1.9	
1950		
1960		
1970		
1980	9.7	
1990		
2000		

2. Using a sheet of graph paper, plot the data from Table 1. Scale your graph from 1940 to 2000 and from one million to 20 million.

3. Project the population data from your graph to the year 2000 and complete Table 1 based on your findings.

4. Make a mural showing your results and place it in your school on a prominent place in your community.

5. The slope-intercept form of a linear equation is $y = mx + b$. A graph of the equation is a straight line with a slope of m and y intercept (o,b).

    a. Graph the equation $y = 2/3x + 1$ beside your population graph.

    b. Explain the differences between the two graphs. Be certain to use the terms arithmetic and exponential in your explanations.

    c. Devise an equation to describe the curve representing Florida's population growth.

6. From Table 1, identify what decades the population of Florida increases at the fastest rate (% increase); the slowest rate.

Optional:

7. If the area of Florida is 38,000,000 and if it takes 1 acre to support a person (food, waste disposal, housing, etc.), how long will it be before Florida is "FULL?"*

8. When Florida is "FULL" of humans, what will be the conditions for other life forms?

9. What role do other life forms play in the support of the human species?

10. Give careful thought to the possibility that your answer in (7) is too large or too small. Indicate what factors might affect your result in either direction.

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*Reference: Brown, L.R. "Fuel Farms: Cropland of the Future?" *The Futurist* 14:16-28, Feb. 1980.

DENSITY

What will the density of your school be for the next 20 years if the population increases at a rate of 10 percent per year?

Here is an example to show you how the students at Jefferson High worked this problem:

1. The current density of Jefferson High is .5 students per square yard. If there is a 10% increase the first year, then there will be .55 students /er square yard. The third year there will be .61 per square yard. Year four will have .67 per square yard, etc.
2. Simply multiply the current year (.5) times 10% (.10), then add the answer to the current year.
3. Have students round off at the nearest hundreth. Thus, answers would be as follows:

Year	Density	Year	Density	Year	Density	Year	Density
1	.50	6	.81	11	1.30	16	2.09
2	.55	7	.89	12	1.03	17	2.30
3	.61	8	.97	13	1.57	18	2.53
4	.67	9	1.07	14	1.73	19	2.78
5	.73	10	1.18	15	1.90	20	3.06

Now, work with your partners and figure out the increase in density that will occur at your school. Make a chart like the one above thea will show your answer.

DISCUSSION:

1. Will there be enough desks in the classrooms to take care of the increase in the next 20 years?
2. If your children are going to school 20 years from now, do you think they will feel crowded?
3. How do you think it will be between classes when everyone is in the hallways 20 years from now?
4. What solutions can you give for this overcrowding problem?

5. Who will have to pay for their school additions?

6. What values are in conflict and what kinds of things must we trade off in order to have more students in school?

STUDENT PROJECTS:

1. Find out how much open land area is converted or changed each year to development in your community (building of houses, stores, roads, etc.). Figure in square miles as well as acres. Make a scale drawing of your community and show how much new development area this represents. At the rate your community is changing, when will all greenspace (forests, swamps, beaches, etc.) be gone?

2. Using the questions listed below, interview members of your neighborhood to ascertain their views about environmental problems. Then use the same questions to interview students in your school. Compare the responses of the two groups. How would you explain the differences?

   a. In your opinion, does an environmental crisis exist today? Why or why not?

   b. What single problem do you think is the most important environmental problem? Why do you think so?

   c. What are the major causes of the environmental crisis?

   d. What are you no doing about the environmental crisis?

   e. What should be done about the environmental crisis?

3. Take a "litter count" in the various hallways and court yards of the school. Keep a frequency count of the types of litter at various locations. What could you as a class or individual do to improve the situation?

4. How many trees are used to make the newsprint for your newspaper in a year if it takes 17 trees to make a ton of newsprint? Suggested approach: Collect the newspaper used in your home for a week.

Weight this and determine the approximate weight of newspaper you would use in a year. How many trees were used in making that amount of newsprint?

5. Hot lunch survey: Get permission to conduct a survey of wasted food in the school cafeteria. Pairs of students (for each type of food served daily) to tally the amounts of food not consumed and milk that is not drunk. Compiled data can be presented graphically to school officials in hopes of cutting down on waste.

Some sample questions you might be able to answer:

a. What foods are most frequently wasted?

b. Does the type of food wasted vary with the time of the year?

6. Select a small area of your school grounds to use as an experiment to test people's reaction to situations of environmental deterioration. Plan ways in which you can make the selected area deteriorate gradually over several days. For Example, you might do the following: First, scatter a few pieces of paper over the area; then, add empty bottles; next add several ugly signs or torn posters, and so on. Be careful, however, not to do anything that will cause permanent harm and do not stray from your planned procedure. (You may have to arrange the experiment in advance with the custodian.) Notice how long it takes for someone to complain or do something about the situation and not who does act. Discuss as a class what happened and why.

7. (This problem is for a student who does not live in a densely populated area and who would have trouble doing a survey type project.) The average American uses 60 gallons of water per day in the home. The percentage breakdown is the following:

Flushing toilets	41%
Washing and Bathing	37%
Kitchen Use	6%
Drinking Water	5%
Washing Clothes	4%
General Household Cleaning	3%
Watering the Garden	3%
Washing the Car	1%

a. Make a circle graph to illustrate the above information.

b. Make a bar graph.

c. Check your family's consumption to see if it appears with these statistics.

8. Americans throw away about 3.2 pounds of rubbish per person a day. Devise your own symbol and key to develop a pictorial graph showing the amount of rubbish disposed of in one week.

9. By the end of 1970 Americans had discarded 50 billion tons of paper and 4 million tons of plastics.

a. If this could be compressed into cubes one foot per dimension weighing 50 pounds each, how many such cubes were created at the end of 1970?

b. What might be a set of dimensions for a space long enough to hold this waste?

c. If 35% of this waste could be recycled (at present, only about 10% of paper is being recycled), how many cubic feet of space would be saved?

d. How many pounds would be returned as a usable product? How many tons?

e. Find out how many rooms the size of your classroom would be needed to contain this waste.

10. If you are in the habit of letting the water run when you brush your teeth, try this plan. The next 5-10 times you brush your teeth, put a container under the faucet and catch the water that would have run down the drain. Measure the amount and arrive at the average number of pints, quarts or gallons that would have gone down the drain. If you brush your teeth on the average of 3 times a day, how much water would you use in 1 day? If there are 4 members in a family, what would such a family waste in 1 day? 1 week? 1 year? If you limited yourself to using just 1 cup (standard measuring cup) of water to brush your teeth, what fractional part would this be of the water that you used when letting the water run? Can you ex-express this savings as a percent?

There are 7½ gallons of water in a cubic foot. How many cubic feet of water would the family of 4 have used in the year? Find the cost of a cubic foot of water in the community in which you attend school. Find the cost of the water used then by a family of four in your community.

11. How much and what kind of litter exists in your community? Can we count every piece of litter in the community to determine the answer to the above question? How can we find a reasonable answer to the question?

A suggested procedure:

1. Get a map of the city and determine sample locations.
2. Establish a unit area for each location. (Example: 100 square foot)
3. Measure to determine the unit area in each location and collect data from that sample area.

WILD DOGS

OBJECTIVE: Students will demonstrate their understanding of the effects of:

Birth rate
Mortality
Birth Control

by using tables and graphs to express population phenomena.

MATERIALS:
Copies of tables 1, 2 and 3
Graph paper
Calculators (optional)

TABLE 1

Breeding Potential of a Free Breeding Canine Population

	No. of Males	No. of Females	No. of Puppies	Total Population
	1	1		2
Year 1	1	1	4	6
Year 2	3	3	12	18
Year 3	9	9	36	54
Year 4	27	27	108	162
Year 5	81	81	324	486

After 5 years of free breeding the humane society started a spay-neuter clinic. The clinic was very successful and was able to successfully neuter half the puppies each year. Let's look at the breeding potential of the population with 1/2 the young being removed.

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TABLE 2

Breeding Potential of a Free Breeding Canine Population with One-half the Offspring Removed

	No. of Males	No. of Females	No. of Puppies	Total Population
Year 5	81	81	324	486
Year 6	243	243	486	972
Year 7	486	486	972	1944

After two years the humane society realized they were not making a significant difference. A scientist suggested they concentrate their efforts on the female puppies. Here are the results.

TABLE 3

Breeding Potential of a Free Breeding Canine Population with All Female Puppies Spayed

	No. of Males	No. of Females	No. of Puppies	Total Population
Year 7	486	486	972	1944
Year 8	972	972		1944
Year 9	972	972		1944

Table 3 clearly shows the effects of removing females from the breeding population. The population stabilizes; and then declines.

Student Activities:

1. Fill in the missing numbers in Table 1 and Table 3.
2. Prepare a "Letter to the Editor" explaining the breeding potential of dog populations. You might want to check with your local humane society/animal shelter to get some figures from your county.
3. Obviously canine populations do not grow as quickly as their potential demonstrates (Table 1). Identify ways animal populations are kept under control.

4. Develop a math formula that describes the population growth in Year 3 and Year 8.

5. One assumption is there are no naturally occuring deaths in the population. Is this realistic? Explain.

6. Suppose the female dogs can only breed for three years. Redo Table 1 showing the effect of female mortality for animals three years old. (Hint: Add another column on the Table.)

7. Express the "revised" Table 1 as a math formula.

8. A humane group has proposed to open a spay-neuter clinic in your county. They claim it will greatly reduce the number of strays. Prepare a short statement to present to the county commission discussing your views on this issue. Assume your county has a population of 10,000 dogs.

CONSEQUENCES

Human population size, like that of other organisms, will be limited by environmental controls. However, by the time the carrying capacity level for humans is reached, the quality of life will be very unpleasant.

A) Overcrowding in human beings leads to increased aggression.

Activity 1: Keeping in mind that the population of this country increased approximately 20% over ten years (1960-1970), ask the students to complete this chart by determining the percentage of increase.

INDEX OF CRIME

1960-1970

per 100,000 population

Offense	1960	1962	1964	1966	1968	1970	% Increase
Murder	5	4	5	6	7	8	
Robbery	60	59	68	80	131	172	
Burglary	500	526	624	708	915	1068	
Larceny ($50+)	282	308	384	457	636	859	

- --Upon completing the computations have students list as many reasons as possible for this disproportionate increase.

- --How many of these reasons could be related to the often heard phrase "population explosion?" (Sociologists indicate that increased crowding results in increased crime rates.)

- --Does it appear that populations do have comfort zones? (Many feel that as a population rises above 40-100,000 in a city that all other disorders such as crime begin to grow at an alarming rate.)

- --What happens when populations go above governmental capabilities? (Higher taxes and more bureaucracy.)

- --What happens when populations are very small? Do they support governments? (Yes and at less cost to the public.)

- --Do you think the crime rates will ever come down again?

- --How much of this increase in crime is related to changing social values rather than density? (Comparing crime rates per 100,000 in a small city versus a large city such as Jacksonville could help answer the question. Large city crime rates have been significantly higher.)

- --Theorize as to the reasons for density-related crime? (Compare the crime in single-family residential areas versus highrise apartments. What does the difference in crime rates have to do with territoriality?

Activity 2:

Draw the United States and Europe on large poster board paper (to scale). Locate at least 10 large cities with a population of at least 1 million on both and mark them with bright red stars. Find the crime rate of each starred city and mark it next to the city in black. Now locate at least 10 smaller cities with population under one million and mark them with a blue dot. Find the crime rate of each blue dotted city and mark it next to the city in yellow. Make a key at the bottom of your posters.

- --Is there a large difference in the crime rates of cities where there is apparent overcrowding? If so, why?

- --Where would you expect the crime rate to be at its lowest? Highest?

--Why is the crime rate higher in crowded areas?

--How would you keep crime rates down without more policemen?

B) Overcrowding in human beings leads to abnormal behavior.

Activity 1: Perhaps narcotics addiction might serve as a resonsably valid gauge for psychological pressures experienced by a population as drugs are most often used for an escape from these pressures.

Have students study and graph the following table:

NARCOTIC ADDITION IN THE U.S.A.

per 100,000 population

1960	1968	1969	1970	% of change
25	32	33	33	37.6

Discuss correlations between the following terms:

caged man
population
tenement
job pressure
peer pressure
rush hour
traffic
urban
stagnant air
garbage
overusage
depression
suppression
aggression

--by contacting a drug rehabilitation program, try to answer the question:

Is the frequency of narcotics addiction greater in big cities?

Activity 2: Mark off an area 6 feet square in the classroom, suggesting that this represents the environment. Have one student (students representing population) enter the space. At intervals of one minute, add students to the area in the manner of a geometric progression (representing population growth: 1, 2, 4, 8, 16, 32, etc.). No student should be in contact with any other student inside the area. Continue until no more students can enter the area. (Note that, as the population of this area increased, each student had less space available for himself.)

--What percentage of the class was able to enter the area?

--How long did it take to fill one-half the area? To fill the entire area?

--Suggest some additional deprivations that individuals would suffer under actual conditions of high density.

--Is there a limit to what the world population can safely become?

--Do you think tension increased as the population grew? Why?

--Does this experiment accurately depict the real environment? Why not? Where would the individuals get their food if they were standing on all the available ground? How much space does each person really need to satisfy his basic needs?

--What kinds of psychological problems would the individual face as the land availability approached the minimum required for each person?

Activity 3: The basic needs of men and animals seem to be approximately the same. These needs include a suitable living area, freedom from stress, territory, and a stable reproductive process. Have your students consider the plight of a caged animal. Few animals readily adapt to this condition. Some reference study or a visit to a zoo will reinforce this understanding.

Discuss a "caged" man. Such things as job pressures, tenement housing, crowded freeways, busy shopping centers, and overused recreation areas may serve to trap a human being.

--What other factors can you think of which tend to "cage" man?

--What influences an animal in its selection of a home territory?

--Do human beings consider the same things when seeking a place to live?

--Is the environment compatible with the needs of man? Explain.

--What adjustments or avenues of escape can man exercise to insure his well-being and survival that lower animals cannot make or use?

--How do rural, suburban and urban environments each suit or thwart the needs of man?

Activity 4: Write letters to various state mental institutions for information on abnormal human behavior. Ask specifically for information concerning the numbers of admissions from large urban areas versus the number of small rural areas.

--Which area showed the greatest incidence of psychological disorders? (Remember to calculate the percent of disorders from each area and do not use raw data.)

--Why is there a greater incidence of disorders in the urban areas?

--How would living in an apartment alter your psychological adjustment?

C) Crowding of human beings results in increased incidence of physicological disease.

Activity 1: The following statistics are offered to the students for contemplation:

DEATHS DUE TO CANCER AND HEART DISEASE

1904-1970

per 100,000 population

Year	1904	1924	1944	1949	1950	1950 (revised)	1970
Cancer	68	87	123	134	137	139	162
Heart	153	169	303	325	327	356	360

As cancer and heart disease are so closely linked to pollution and other modern day "additives," it is felt that these two ailments may well act as baromenters for physiological disorders related to the population explosion. Have students graph the above chart. Have students speculate on these figures without having specific information on the nature of these diseases. Discuss population pressures and its effect upon general health.

--How do you think the rates of these diseases in rural situations would compare to those in the city? (Find out.)

Activity 2: Have an official from the County Health Department come to your classes about contagious diseases. Have him discuss their ease of transmission in crowded areas vs. uncrowded areas.

--Is your county beginning to get large enough so that conta-disease epidemics will become more prevalent?

--Is the crowding at school more conducive to disease transmission?

--Compare the spread of contagious diseases through an apartment building with the spread of the disease in a single-family residential area.

--Is our population becoming large enough for a country-wide spread of diseases such as Asian flu?

--What can be done to minimize the spread of these contagious diseases?

D) As the demand for space increases, competition among human beings will increase for those spaces.

Activity 1: Remove one chair each day from the class and note the student behavior as competition increases for the remaining chairs. The chairs would have to be removed before class and unknown to the students. A small corps of close-mouthed students might act as statisticians for the experiment. However, the resulting antagonistic attitude in the cehaired group will serve as testimonial for this concept, regardless of accurate records.

--Is this competition for space a real phenomenon? (Yes. Competition for land, parks, lots to build on, etc. is increasing every day.)

--Do you think national and state parks will ever regulate attendance on the basis of a lottery? (Many already do and in the next 10 years several of our national parks will go on this system.) Is this competition for space?

--Can you think of any areas where man competes for the use of land such as residential vs. agricultural? (There are many areas like this all over the country.)

--In Europe there are cities where people have to wait to get a place to live before they get married. What do you think is happening to the couples average age at the time of marriage? (Yes, it's going up.)

--As the competition for space increases, what do you think the future will be like? (Try to answer this question accurately without going too far out. Maybe the students could write a paper on this.)

--What will happen to the cost of space as it gets scarce? (Go up.)

Activity 2: Have a group of students compose a skit with the basic theme "Too Many People Live here." Have them include the problems in everyday life that they encounter (competition for food, materials, space, water, etc.)

Activity 3: Tell the students they all own 5 feet of the beach. Have them write a paragraph on what they would do with it. Then tell them people are moving into the area and 2 feet of land will be taken away from their 5.

Have them write a second paragraph expressing their feelings about this action. Next, one more foot is taken away from each student due to more babies being born in the beach community. In this last paragraph have them write what they would do about this last action.

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POP QUESTIONS

OBJECTIVE: Students should be able to recognize a growth curve in an unchecked population, and to interprete the slope of a graph.

MATERIALS: Copies of the Graph (1-4)

DISCUSS:

1. What environmental conditions would be responsible for the stationary phase? (Graph 1) For the decline phase?

2. What phase of the growth curve is the human population now in? (Graph 2)

3. Compare the human population graph with the number of species becoming extinct. Why, what factors, cause these to be related? Can this trend be reversed?

4. What happens to an area when it is overpopulated? How many deer can the range support in 1940?

5. Explain the fluctuation of the snowshoe hares (Graph 5) in tye years 1900 to 1905. Were the rabbits on the incline or decline phase?

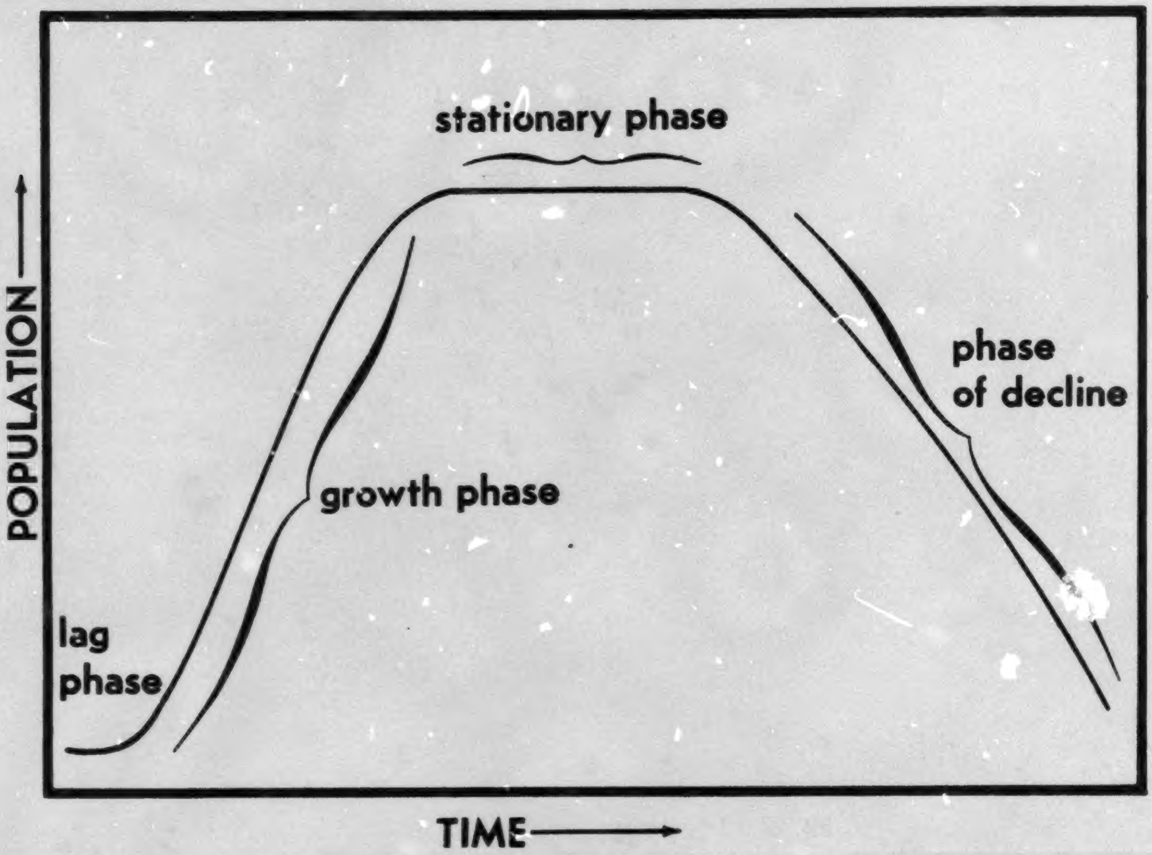
6. Collect from the newspaper or magazine other examples of the growth curve.

7. Why is there a "dip" in the curve (Graph 6) for the plague and not a dip for World War I or World War II?

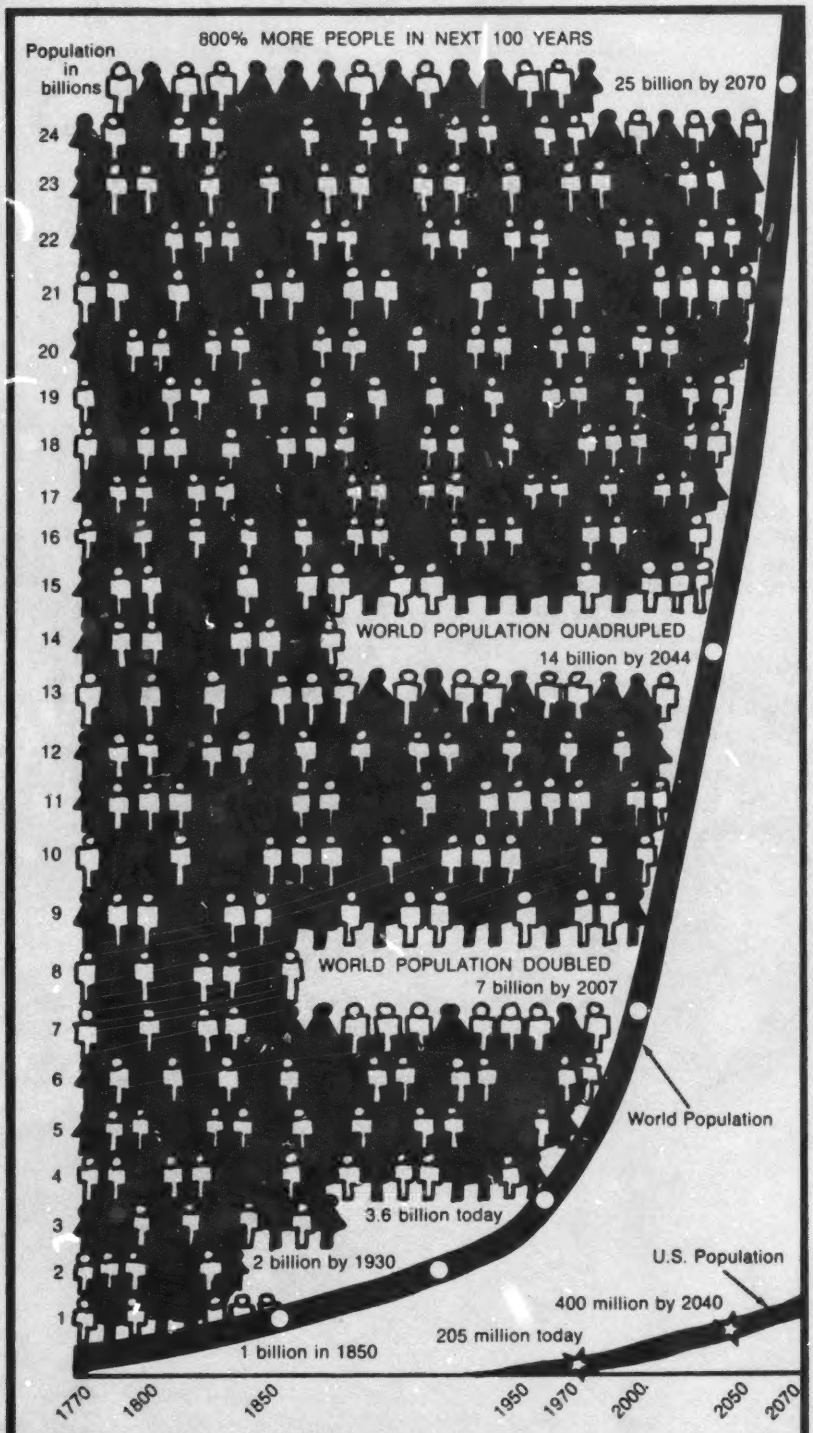
Scholastic Teacher
Science World   Science Focus   Transparency Master No. 9   October 14,1966

GRAPH 1

POPULATION GROWTH



GRAPH 2



World population is skyrocketing

It took more than a million years for our population to reach one billion in 1850. Today, we have 3.6 billion. Estimates are for 7 billion in 30 years. Some scientists grimly warn our earth cannot sustain that many people.

National Wildlife J.F. 1971

GRAPH 3

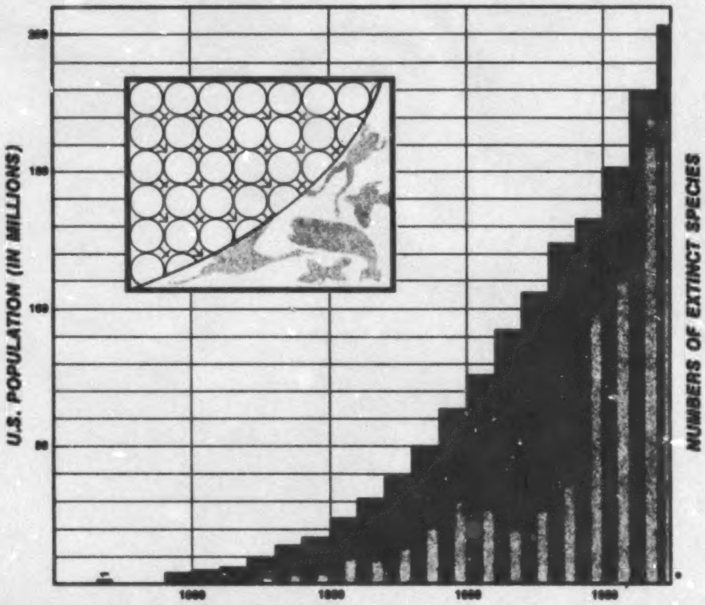
National Wildlife

WILDLIFE OMNIBUS

People vs. wildlife

FOR 58 OF THE 109 species currently listed as endangered in the U.S., the chances of becoming statistics on this graph have been reduced in recent months by a U.S. Fish and Wildlife Service program. To cope with the problems of individual species, 47 highly specialized "recovery teams" have been established.

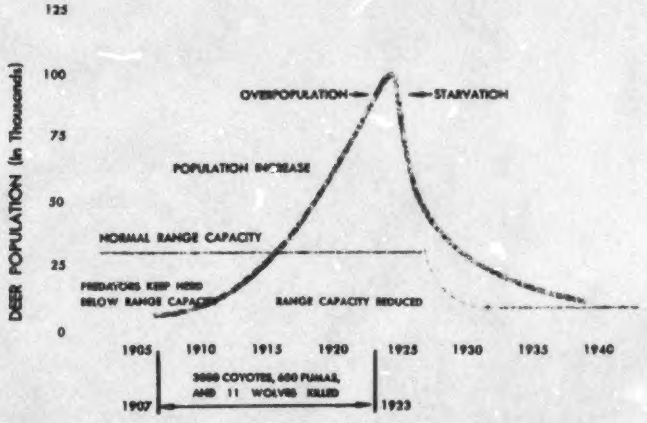
According to David B. Marshall of the Office of Endangered Species, three criteria were used to select target species: the urgency or degree of threat to the creature, such as that faced by the California condor; the need to coordinate existing programs, such as with the eastern timber wolf, and the need to acquire land to protect the species. On each team, experts with a wide variety of backgrounds have been included. A real estate specialist, for example, has been assigned to the Mississippi sandhill crane's team, since land acquisition is vital to the bird's future. During the next fiscal year, 11 more teams will be established.



Since 1750, 798 species of birds, mammals, fish, plants and mollusca have become extinct in the U.S. This graph illustrates the relationship between the number of extinctions in each decade (light brown) and the nation's population growth (dark brown). Compiled by Paul Opler of the U.S. Office of Endangered Species, the graph suggests that, if the rate of extinction for the 1970s continues at its present pace, at least 150 to 200 more species may be lost in this country before the year 1980.

GRAPH 4

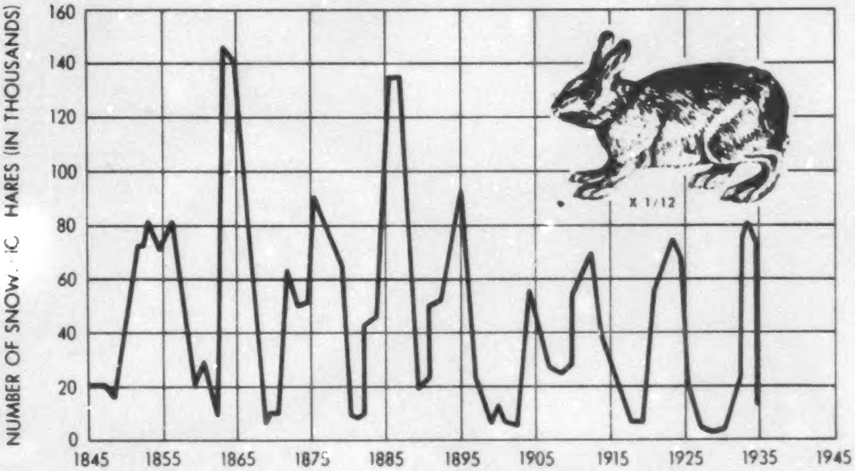
Figure 19. Changes in size of the Kaibab deer population resulting from a reduction in the number of predators. Note the lowered capacity of the range as well as the changes in the deer population. (Adapted from *Life, An Introduction to Biology*, by Simpson, Pittendrigh and Tiffany, © 1957 by Harcourt, Brace & World.)



GRAPH 5

Rand-McNally
BS. CS. Green - 1966

Graph of changes in the snowshoe-hare population, Canada (based on skins traded at Hudson's Bay Company posts).



ELECTRIFYING GROWTH!!

OBJECTIVE: Student will be able to graph growth in electric consumption and generation in Florida using the concept of exponential growth.

MATERIALS: Copies of Tables 1 and 2
Graph paper

TABLE 1

Electric Utility Industry Net Generation
In Florida 1970-79
(Millions of Kilowatt Hours)

Year	Total	% Increase
1965	30,273	
1966	34,404	
1967	38,176	
1968	43,775	
1969	48,840	
1970	55,469	
1971	60,662	
1972	67,407	
1973	76,452	
1974	74,836	
1975	77,825	
1976	80,957	
1977	86,803	
1978	91,514	
1979	94,090	
1980 (Est)	97,000	

NOTE: Net electric generation equals gross generation less kilowatt hours consumed out of gross generation for station use.

Source: State of Florida, Public Service Commission, Statistics of the Florida Electric Utility Industry, 1979.

Florida Statistical Abstract 1981

TABLE 2

Electrical Utility Industry: Net Generation by Type of Fuel in Florida 1965-1980 (Millions of Kilowatt hours)

Year	Total	Coal	Fuel Oil	Gas	Nuclear	Hydro
1965	30,273	5,399	17,019	7,556	0	298
1970	55,469	11,394	25,829	17,954	0	290
1975	77,825	12,592	44,045	12,586	8,370	232
1979	94,090	18,301	45,034	15,113	15,396	247
1980 (Ext.)	97,000					

Source: State of Florida, Public Service Commission, Statistics of the Florida Electric Utility Industry, 1979, and previous edition.

Florida Statistical Abstract 1981

Tables 1 and 2 provide an excellent picture of the growth in electrical consumption in the past few years. Table 1 shows an increase in total consumption from 30,273 to 97,000 MM KWL (320%) in just 15 years! Table 2 shows Florida is actually using more oil and gas to make electricity than it was in 1975. What's going on?

1. Complete Table 1 by calculating the percent increase in Florida consumption for 1965-1970
   1970-1975
   1975-1980

2. Graph the electrical energy generated for 1965, 1970, 1975 and 1980. Plot the best straight line among the data points. Write the equation for the line beneath it.

3. Complete Table 2 by estimating the amounts of fuel by type needed to generate the estimated 97,000 MM KWH used in 1980.

4. Based on your analysis in (3), extend the data in Table 2 to the year 2000.

5. On a single sheet of graph paper, chart Florida's consumption of fuels used to generate electricity in 5 year intervals from 1965 to 2000.

5a. If an 800 megawatt electrical generating plant will generate 4 billion KWH/year, how many of these generating plants will need to be built between 1980 and 2000 if the trend continues?

5b. If an 800 megawatt electrical generating plant costs 1 billion dollars, how much will need to be spent on power plants between 1980 and 2000 if the trend continues?

5c. Where will we put these plants?

5d. Can you imagine a way that this money might be better spent?

(a) List the fuels that are exhibiting negative growth.

(b) List the fuels that are exhibiting arithmetic (linear) growth.

(c) List the fuels that are exhibiting exponential (geometric) growth.

6. Write a paragraph describing Floria's environment if your projections become true.

7. Write a paragraph or use art or other non-verbal form of communication to develop alternatives to your projections.

IS MORE BETTER?

This lesson examines common assumptions about increasing energy use and the quality of life. Students examine the pervasive use of energy in modern life by measuring the energy consumed in producing disposable materials. Students research societies that use less energy per capita than the United States. A final research activity directs students' attention to the possibilities of a more energy-efficient future.

OBJECTIVES: Students should be able to:

* Examine advertisements for examples of energy intensive ways of living and identify causes of increasing electricity use.
* Use a checklist to formulate generalizations about energy and the quality of life. Critique these generalizations on the basis of evidence they have collected.
* Construct energy flowcharts to determine how much energy is used in producing disposable containers.
* Research and report how other societies use energy.
* Collect data to develop predictions on growth patterns in their city or town.

MATERIALS: Pictures, maps and artifacts of hometowns 30 to 60 years ago. (Ask students to collect them in advance.)

Pictures of cities today.

Copies of Student Handout "Growth and the Quality of Life."

PROCEDURE: A. Divide the class into groups. Have each group find pictures or advertisements in magazines and newspapers of those items they think would make good gift items.

Activity 1: Tack these pictures or ads on the bulletin board. Then have a member of each group list the items and take the list back to the group. Ask group members to evaluate the items in light of the questions below:

1. What do these ads and pictures tell about the way energy consumption is growing?

2. Why do people want these products? Do people need them?

3. Read some of the ads closely. What emotion is each appealing to?

4. How would an energy-conscious consumer respond to many of these ads?

B. Divide the list into several categories:

Electrical products
Non-moving products
Manually-operated products
Battery-operated products

Have students estimate the percentage of items that use electrical energy.

Activity 2: Give students one or two class periods to read and mark their lists, "Growth and the Quality of Life." After everyone has completed marking, group the students into pairs. Tell each pair to write generalizations about contemporary American life and about conservation of energy and its effects on the quality of life. Each team should:

1. Write at least five generalizations.

2. Defend or criticize each generalization about conservation and the quality of life on the basis of the evidence in the list.

It might be well to make the first generalization together as a class. Make sure the students understand the meaning of the term generalization.

Possible answers include:

a. Generalization: American homes have more uses for electricity today than thirty years ago.

Critique: This seems to be substantiated by the growth in electrical appliances.

b. Generalization: Businesses have always given consumers what they wanted in the way of convenience.

Critique: Partially substantiated by the number of appliances and conveniences listed. The generalization is debatable because what people want may be dictated by what is offered them. True wants and needs can be blurred by advertising.

c. Generalization: People need all the appliances and conveniences they can afford in order to live a comfortable life.

Critique: Not substantiated. Americans are human and have lived rather well without a maximum of conveniences. The generalization is debatable because what is meant by a decent life or quality of life is open to question.

d. Generalization: Buying machines to help do household and personal chores has been important to Americans over the last thirty years.

Critique: This seems to be substantiated by the lists.

e. Generalization: Conserving energy will mean doing without things that make life meaningful and decent.

Critique: Not substantiated. Conservation also means using machines that are more efficient. It can include recycling some materials, using lighter cars, and insulating homes. Some of these advances could make life more comfortable and convenient.

Activity 3: Assign three groups of students to research and report on people whose lives are less energy-intensive than Americans. One group could investigate people living in underdeveloped countries. Find out:

a. How are human and animal muscle power used in these countries?

b. How do these forms of energy conserve fossil fuels?

c. What activities by these people are likely to change if the society modernizes? What activities will probably remain unchanged?

d. What renewable energy sources are available to people in this society? (Examples: water, sun, wood, animal and human waste.)

Another group should investigate a developed country that uses less energy than the Uninted States, such as Sweden or West Germany. (An excellent reference is "Efficient Energy Use and Well-Being: The Swedish Example." Lee Schipper and Allan V. Lichtenberg, Science, December 3, 1976, Volume 194, Number 4269, pp. 1001-1013.)

What do other countries do to conserve energy and yet maintain a high quality of life? Encourage students to investigate the use of smaller cars, cogeneration, mass transportation, recycling, and more energy efficient factories.

The third group could investigate ways the United States could use less energy and yet maintain a high quality of life. Investigate microcircuitry, new engines that are more efficient, congeneration, recycling, installing better insulation, etc.

Extending the Lesson: Ask students to bring in pictures, maps and artifacts of their town 30 to 60 years ago. Then divide the class into three groups--(1) those to look at technological changes from then to now, (2) those to examine growth patterns, and (3) those to identify the changes in energy use and consumption over the years. Have the groups consult with each other to exchange information whenever they need to. (You may wish to assign a small group of students to act as research assistants to find data and pictures. This group should work closely with the town librarians and histroians as well as the school librarian.)

Hold a "town meeting" to give students a focus for their reports. Tell students that the evidence shows that our city today is consuming more energy in part because the population has grown. Ask" "What will the population increase in another fifty years do to energy consumption?" Explain that the constant doubling of the population can be demonstrated by simple mathematics. Suggest this proposition: If a town has 1,000 people and a constant doubling time of 25 years, how much larger will the population be in 100 years? Will it be twice as large? Three times? Ten times or sixteen times? Students should compute the answer and the rate of growth. (Refer to activities on doubling time and growth rate.)

Answer: 16 times larger

at 25 yrs.	2000
at 50 yrs.	4000
at 75 yrs.	8000
at 100 yrs.	16000

2.8% growth rate

$R = \frac{70}{T_2}$ $\qquad T_2 = 25$

$R = \frac{70}{25} = 2.8$

If the population is 16 times larger in 100 years, ask: "How much more energy, food, jobs, and power plants will be needed in the future (for the city)? What can we use as alternate energy sources if our finite energy sources become scarce?" (Accept all suggestions.) Have students do similar problems relating to energy-using consumer goods.

NAME____DATE__________

GROWTH AND THE QUALITY OF LIFE

	1	2	3	4	5	6
NAME OF ITEM	Number of each item you have in your home	Check if item was in your house at the time you were born	Check if item was available when your oldest parent or guardian was born	Check if this item is considered a necessity by your family	Place an "O" if this item is considered a luxury by your family	Check if you think the item is necessary for you to live the "good life"
power lawnmower						
battery operated sphygmomanometer						
automobile						
motorcycle						
flush toilet						
snowmobile						
power boat						
C.B. radio						
Electric:						
coffee maker						
skillet						
mixer						
toaster						
toaster oven						
microwave oven						
food processor						
potato peeler						
toothbrush						
shaver						
hot dog cooker						
grill						
rotisserie						
wok						
egg poacher						
streamer						
fondue						
popcorn popper						
yogurt maker						
crepe maker						
tennis ball return						
cookie gun						
vibrator						
whirlpool						
heated swimming pool						
apple corer						
self-cleaning oven						
refrigerator						
freezer						
garbage disposal						
clothes washer						

NAME OF ITEM	1	2	3	4	5	6
Electric:						
water softner						
hair dryer						
food grinder						
bottle sterilizer						
baby food dish						
hamburger cooker						
saw						
hot plate						
ice cream maker						
waffle iron						
juicer						
teakettle						
knife sharpener						
movie projector						
leaf sweeper						
hedge trimmer						
meat slicer						
night light						
grass clippers						
stapler						
typewriter						
plant lights						
shoe polisher						
rake						
car washer						
insect sprayer						
roaster						
hair curlers						
rotating TV antenna						
floor polisher						
sandwich grill						
sauna heater						
scissors						
rug shampooer						
can opener						
slow cooker						
plastic bag sealer						
cooler						
knife						
fan						
compost shredder						
water heater						
mixer						
iron						
garage door opener						
curling iron						
deep fryer						
humidifier						
dehumidifier						
dictating machine						
dishwasher						
lawn edger						

NAME OF ITEM	1	2	3	4	5	6
Electric:						
curtain rods						
bun warmer						
dutch oven						
organ						
piano						
doorbell						
skin machine						
radio						
clock						
griddle						
meat grinder						
tape deck						
stereo						
black & white TV						
color TV						
Whirlpool bath						
socks						
handwarmer						
bed springs						
sander						
engine heater						
food dehydrator						
wall paper steamer						
train						
telephone						
furnace						
sewing machine						
calculator						
vacuum cleaner						
lawn clippers						
drill						
blanket						
freezer						
guitar						
telephone answering system						
time						
dental hygiene set						
welder						
attic ventilator						
back messagers						
golf cart						
air conditioner						
bread mixer						
broiler						
sunlamp						
(OTHER ITEMS?)						
____________						
____________						
____________						

BUILDUP OF CARBON DIOXIDE IN THE EARTH'S ATMOSPHERE

The level of carbon dioxide buildup in the Earth's atmosphere as a result of the worldwide consumption of fossil fuels appears to be on an exponential growth curve. (see Graph)

The primary concern about the buildup of CO_2 in the atmosphere is that it will lead to global warming - the "greenhouse effect." A recent review of the literature concludes that - with a 2 percent annual rate of increase in worldwide consumption of fossil fuels (that is, essentially zero growth in per capita use of fossil fuels at the current world population growth rate) - the average global temperature would increase by 0.5 to 2.5°C from the carbon dioxide buildup in the next 50 years. A degree of temperature matters little in a day's weather, but as a change in a global average, it means a lot.

It is difficult to fully predict the effect of this global temperature rise. Some currently semi-arid regions would probably experience increased rainfall and some northern areas would probably enjoy longer growing seasons. But, some important food producing regions could be seriously damaged. If temperate-zone climates were to warm by some 2.2°C, then some atmospheric models predict that polar temperatures would rise by up to 8.8°C. That could well prove sufficient to change the temperature structure of the oceans, thus causing a major rise in sea level. The oceans contain some sixty times as much CO_2 as the atmosphere, and each degree of warming could release enough CO_2 to increase the atmospheric content by 3 percent, leading to still further warming.

The syndrome of human-generated CO_2, polar warming, deep ocean warming, and release of oceanic CO_2 could bring rapid, unplanned changes. There are enough fossil fuels available to produce a tenfold rise in CO_2 levels. Depending on the degree of polar warming and ice cap melting, the rise in world sea levels could exceed a hundred feet.

Attendant to increased fossil fuel burning (especially coal in electrical power production) is the increased atmospheric load of sulfur dioxide and nitrogen oxide. The result is "acid rain" in certain industrial regions due to the production of mild solutions of sufuric acid in the atmosphere. The phenomenon leads to widescale alteration of environments and habitats.

PH is a measure of rainfall acidity. The pH scale measures both acidity and alkalinity. The scale ranges from 0 (the most acidic) to 14 (the most alkaline) with 7 as neutral. The lower the number, the more acidic is the solution and the higher the number the more alkaline. Rainfall theoretically has a pH of 5.6. Milk has a pH of 6.5, and orange - 4.6, tomatoes - 4.2, vinegar - 2.2, baking soda - 8, and ammonia - 12. As the numbers get higher, the solution becomes more alkaline and as they get lower, more acidic.

Activities:

1. Using the graph, have students predict what CO_2 levels will be in the year 2000 at a 4% use rate. Prédict for the year 2020 CO_2 levels at a 2% reate of increase and a 4% rate.

2. Widescale changes may result from small average increases in temperature. Any change in the system sets off chains of events that are related.

   Assign students to research the effect of increases in atmospheric temperatures on snow and snow accumulation, average annual rainfall, the reflection of sunlight off snow and ice, the length of crop growing seasons, water levels in reservoirs, etc. (See attached Bibliography)

3. Have students develop scenarios to describe life on this planet under the conditions plotted in #1.

4. Discuss the effect of the rapid destruction of the world's great rain forests on natural CO_2 absorption. (Not only are the plants no longer available to absorb atmospheric CO_2, but clearing forests represents an additional source of CO_2. Trees chopped down are usually burned or left to decay; in either case there is CO_2 release.

5. If the world's climate warms up, a consequence could be the melting of polar ice and rising sea levels. Give students a map of Florida with elevations and ask them to shade areas that would be flooded by a one foot rise in sea level. Then, shade flooded areas at two foot and 3 foot rises.

6. Do an experiment with ice cubes melting. Record the time required for a cube to melt at 4°C(39.2°F) and 21°C(70°F) and 23°C(73.4°F).

7. Contact your county environmental control director and ask for air quality monitoring data for your region. Contact the Florida Electric Power Coordinating Group, 402 Reo St., Suite 214, Tampa, FL. 33609 for statewide figures on acid rain.

   Ask your local environmental control agency to set up air monitoring equipment near your school and share the information with your class.

8. Acid rain is becoming a problem in Florida. Have students collect rainwater samples to determine pH. Acidic rain is that below pH 5.6. However, a variety of natural

factors contribute to acidic rainfall, such as lightning, volcanos, bacterial activity, forest fires, etc. Care should be taken in interpretation. Select a local body of water and have students begin keeping pH records for the lake or pond. (A variety of inexpensive methods are available for determining pH. Write HACH Chemical Co., P.O. Box 389, Loveland, Colorado 80537, or call toll free 800/525-5940). The pH scale itself can be used to illustrate exponential quantities. Students can begin water sampling activities and wildlife inventories to use for comparison to monitor from year to year. Again care should be exercised in interpretatin as blue-green algae, fertilizers, copper sulfate control agents, ground water discharge, stormwater run-off, etc., could all affect the lake's pH. Most natural bodies of water range from pH 4 to pH 9 and are often basic (pH above 7) due to the presence of carbonates and bicarbonates. Each species of life has a limited range of pH in which it can survive. This activity can incorporate many ideas to illustrate the interconnectedness of nature and the complex issues involved in CO_2 buildup and acid rain.

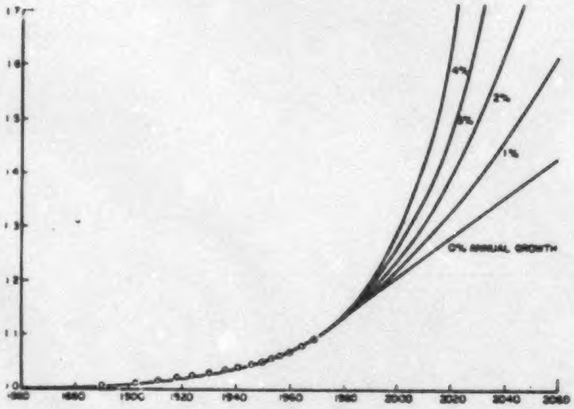


FIGURE 1

BUILDUP OF CARBON DIOXIDE IN EARTH'S ATMOSPHERE

Historical data and projected concentration of carbon dioxide in the atmosphere (pre-industrial level = 1.0) for different assumed growth rates of fossil fuel combustion. The curves are based on the assumption that 50 percent of the carbon dioxide released stays in the atmosphere. Note that, even at current rates of fossil fuel consumption, the excess carbon dioxide concentration due to a man's activities will approximately double between 1970 and 2000.

October 1977 Bulletin of the Atomic Scientists

(Graph taken from von Hippel and Williams, 1977. "Toward a Solar Civilization." The Bulletin of the Atomic Scientists, October, p. 12.)

BIBLIOGRAPHY

CO_2 buildup in the atmosphere

Abelson, P.H. 1977. "Energy and Climate". Science 197: 941..

Broecker, W.S. 1975. "Climatic Change: Are We on the Brink of a Pronounced Global Warming?" Science 189: 460-463.

Kerr, R.A. 1977. "Carbon Dioxide and Climate: Carbon Budget Still Unbalanced." Science 197: 1352-1353.

Siegenthaler, U. and Oeschger, H. 1978. "Predicting Future Atmospheric Carbon Dioxide Levels." Science 199: 388-395.

Woodwell, G.M. 1978. "The Carbon Dioxide Question." Scientific American (Jan.): 34-43.

COLONIES IN SPACE

OBJECTIVE: To investigate the relationship between population growth and limited resources.

In 1976, the world population was estimated to be 4,100,000,000 with an annual growth rete around 2.1%. At that rate the world's population will double in 33 years. Assume that just prior to this, around the year 2000, we will possess the technology and national will to begin constructing colonies in near space. These colonies would derive their resources from mining the moon and capturing sunlight. An artificial environment will be created, one that is self-sufficient and dependent on the planet earth for nothing more than the initial population of plants and animals and microbes.

PROCEDURE: Students are to design a self-sufficient colony in space. They must decide on the optimum size of the colony, the kinds of plants and animals needed, government, recreation, pollution control, defense and so on.

STRATEGY:

1. Divide the class into groups of three. Each group must research alternative solutions to the various needs of the colonists. For instance, one group could research how many people should be sent to establish a self-perpetuating colony. Another could try to find how much oxygen does a single diatom or algae cell or grass plant produce. What is the size of the green space needed to produce food and oxygen and recycle carbon dioxide? How much electrical energy is needed per individual? What size solar collectors will be needed to provide electrical power for the needs of the colonists? (Most of this has been worked out. For example - Using modern techniques, it is estimated 100 acres could supply the needs of 10,000 colonists.)

   Cattle are rather wasteful at converting feed to beef. They need over twice as much feed as rabbits to produce a pound of edible meat.

   Another source of protein could be fish. In a weightless space farm, it may be possible to raise fish without water. Without gravity to collapse the gills out of water, a fish might easily "swim" through an atmosphere of 100 percent humidity, keeping comfortably moist.

   On earth variations in crop yield are due to weather, weeds, insects, rodents and disease. Weather simply does not apply in space, assuming there are no breakdowns in the systems for temperature and humidity control.

(Heppenheimer, 1977) (See References)

2. Each group provides their findings to the rest of the class. Taking this data plus estimates, each student should design a colony. They should imagine and draw its shape, dimensions, and orientation relative to the earth, sun and moon. They should design living quarters, power plants, gardens, public areas, etc. Students should be encouraged to be as imaginative as they wish while staying within the limits set for resource needs.

3. In a classroom discussion, students can share their ideas and the class reach a synthesis of the best concepts and designs. A single space colony design can emerge from this brainstorming.

4. Students can now "operate" their space colony. Choose a day to dress in the manner of a colonist. Set up investigations to test the assumptions made. For instance, set up aquaria to grow algaes, fish, snails, etc. Alter factors to obtain the best yields. Raise rabbits and calculate amount of food that can be derived from eating the rabbits vs. the amount of feed required. Compare this to yields in other kinds of animals. Measure human respiratory rates and oxygen consumption. How much oxygen would be needed per day if all the colonists were school kids? Create space art, write songs and poetry like a colonist.

5. At this point, assume the colony starts to grow in size at 1% a year. Have students calculate what effect this will have on the colony's resources. What could the colonists do to establish a new balance? What could the colonists do to control their population growth?

6. Students could discuss the effect of space colonies on the earth's resources and population.

REFERENCES:

1. Bradfield, R. 1972. "Multiple Cropping: Hope for Hungry Asia." Readers Digest (Oct.): 217-221.
2. Brand, S. ed. 1977. Space Colonies. Penguin Books, New York.
3. Fowler, John W. 1975. Energy-Environment Source Book. National Science Teachers Association, Washington, D. C.
4. Hagler, T. 1976. "Building Large Structures in Space." Astronautics and Aeronautics 14: 56-61.
5. Heppenheimer, T. A. 1977. Colonies in Space. Stackpole Books, Harrisburg, PA. Also available in paperback (Warner)
6. Heppenheimer, T. A. 1979. Toward Distant Suns. Stackpole Books, Harrisburg, PA.
7. Holbrow, C., and Johnson, R., eds. 1977. Space Settlements - A Design Study. Report of the 1975 NASA/ASEE summer study on space colonization. NASA SP-413. Available from Superintendent of Documents, U.S. Government Printing Office, Washington, D.C.
8. Lapedes, Daniel N., ed. 1976. Encyclopedia of Energy. Mcgraw-Hill Book Co., New York.
9. O'Neill, G. K. 1977. The High Frontier. Morrow, New York. Also available in paperback (Bantam)
10. Wuenscher, H. F., 1972. "Manufacturing in Space." Astronautics and Aeronautics 10: 42-54.

Also see: "Food and Agriculture." Special Issue, Scientific American, September, 1976.

To understand the effect of arithmetic growth and exponetial growth on living populations, students need an understanding of the resource requirements of organisms. Specifically, they should be aware that much of the energy available at different levels of an energy pyramid is lost to the next population up the scale. Consequently, a large resource base is often needed to supply even a relatively small population. Disruptions in numbers at any level can have far-ranging consequences.

The following activity should be used as an adjunct to studies of exponential growth in living systems.

This activity, taken from the March 1973 issue of Science and Children involves the entire class. It is an enjoyable way to help students understand energy use in nature, specifically the energy pyramid.

JELLY BEAN ECOLOGY

The concepts dealing with ecological processes are perhaps among the most difficult to deal with in the calssroom. One of the reasons seems to be that the classroom is "not where the action is." The "action" is all around us in varying degrees but not in the hard seats in a classroom.

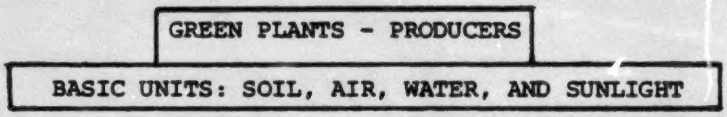
The ideal classroom for developing these ecological concepts would be in an outdoor education area where the processes could be viewed and studied firsthand. Most teachers charged with the responsibility for building basic ecological generalizations find it difficult to move the classroom to this ideal situation.

Why not do the next best thing and "make believe " through role-playing? One ecological concept that can be easily and enjoyably handled through role-playing is the food cycle or more specifically the "energy pyramid" that exists in nature (and a process in which man plays a dominant role).

THE CONCEPT

The "energy pyramid" begins with the basic units of energy that are found in the soil, air, water, and, of course, the radiant energy from the sun.

These basic units of energy are capable of being used by only those living things equipped with the proper facilities for converting the basic units of energy into living matter, namely, the green plants or producers.



Green plants store most of this energy in the form of plant tissue in the leaves, roots, and stems. The only way these units of energy can be relaesed is through another facility, the chemical breakdown of this plant tissue through the digestive system of animals that eat plants, namely the herbivores. These animals can be considered as the primary consumers since they alone are able to retrieve energy directly from the producers, and include insects, mice, rabbits, etc.

*"Jelly Bean Ecology" was written by Robert Deem.

At this point the depletion of energy units begins to be noted. The total energy stored in the plants in the form of leaves, roots and stems is not completely transferred to the primary consumers. Several reasons can be listed for this incomplete transfer:

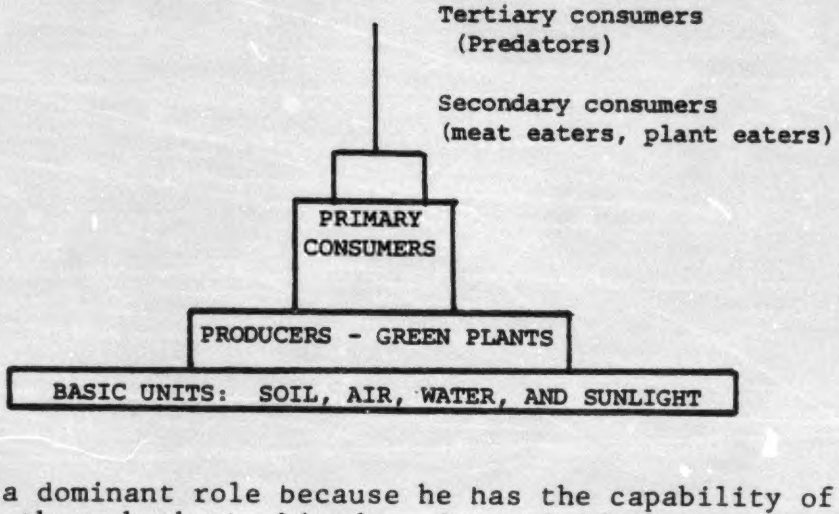
(1) incomplete digestion of plant tissue by the consumers,
(2) incomplete consumption of the plant by the consumers (some roots and stem parts are left in the ground),
(3) some of the energy taken in by plants is needed to carry on the activity of the plant.

It is estimated that only 10 to 20 percent of the plant's energy is accessible to the planter eater.[1]

The energy transfer continues through the action of the animals that depend entirely upon the flesh of other animals for food. These are called the secondary consumers and include such animals as owls (which feed on mice), insect-eating birds, insect-eating insects such as the praying mantis, frogs, etc.

At this stage, if only 10 to 20 percent of the primary consumers energy is passed along to the secondary consumers, only 1 or 2 percent of the original energy taken into the green plants is passed along to these secondary consumers. This means that the range or area hunted by these animals must be fairly large to supply the energy they need for growth and dvelopment.

Topping the pyramid are the tertiary (third order) consumers such as eagles, wolves, alligators, etc. These predators must hunt over a wide range in order to obtian the necessary energy for their survival because the amount of energy transferred to them from the original plants amounts to only .1 percent of the total energy available in the plants.



Man plays a dominant role because he has the capability of controlling all levels through the herbicides, insecticides, and other pesticides as well as the thousands of acres of soil that have been covered by concrete and asphalt.

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[1] Woodwell, George M. "The Energy Cycle of the Biosphere." Scientific American. p. 67. (September 1970).

THE ACTIVITY

To involve students in this basic ecological process requires little more than a bag of jelly beans, some plastic bags, and a class of willing students.

Place 20 jelly beans in each of twenty plastic bags. Each jelly bean will represent one unit of energy that is available from the soil, air, water, or sunlight.

Select or solicit 20 volunteers from the students to act as the "producers" or green plants. Each "plant" is to receive one bag of 20 units of energy (jelly beans). Some discussion of how green plants can make use of soil nutrients, water, air, and sunlight can be reviewed at this time.

Each "plant" will have to "store" 15 units by leaving 15 jelly beans in the bag. These fifteen units would be the energy stored as leaves, stems, and roots of the plants. The remaining 5 units may be eaten by the "plants" since these would be used by the plant for respiration and its normal activity.

A little arithmetic will show that now, instead of the 400 units available originally, there are only 300 units available, 15 from each plant.

Select or solicit four volunteers to act as "primary consumers" or herbivores that live on the energy obtained entirely from plants. Direct each of them to take an empty plastic bag and go collect a total of 60 units from the "plants." They may do this in any kind of combination, but note that each plant must retain three units.

The 12 units taken from each plant represent 60 percent of the original 20 units of energy taken in by each plant, considerably more than the percentage that would be taken in by the plant eaters in the natural setting.

This activity will provide each "plant eater" with 60 units of energy and leave each plant with three units. These three remaining units represent the roots and part of the stem still in the ground from which new growth will come and provide additional energy for the primary consumers.

Most plant eaters require large amounts of energy for activity so those students who have the roles of "plant eaters" may eat 30 units (jelly beans) leaving the other 30 units stored. These in nature would be stored as flesh, bone, fat, and internal organs.

Choose two students from the remainder of the class to act as "secondary consumers" or the meat eaters that have a diet of insects, mice, and other animals. These might be birds, snakes, spiders, or cats.

Direct these "meat eaters" to each take a plastic bag and collect 60 units of energy from any of the "plant eaters." In nature this could mean a fight over a catch that one of the flesh=eating animals has made or it could mean sharing a catch.

Each "meat eater" now has 60 units of energy. Thirty of these units can be used up (eaten) but 30 must be left in the bag as stored energy.

Finally, the predator (chosen from the remainder of the class) such as a hawk, eagle, or wolf, comes into range. Of the original 400 units available in the plants only 60 now remain. The predator collects these remaining units.

If we assume that a large animal such as a wolf requires 30 units of energy just to carry out such activities as hunting, this leaves 30 units for growth. If the predator requires all 60 units for activity, then no growth can take place. This could mean a stunted animal, no reproduction, susceptibility to disease, or other weaknesses. Or the predator might attempt to extend his range to include more soil, water, air and sunlight, and consequently more basic units so that he would survive and remain healthy.

Because of the many and varied avenues that can be taken once this concept of energy use in nature has been firmly conceived in the students' minds, the following questions suggest the possible direction a follow-up discussion might take.

Questions for thought and action:

1. How can the soil continue to provide energy to the green plants? (Certain types of animal and plant life act as reducers which break down dead plant and animal matter into nutrients that seep back into the soil. These would include bacteria, fungi, earthworms, and fly larvae.)

2. What parts of the pyramid are affected by man's use of insecticides to kill grasshoppers, aphids, and beetles in his gardens? (Everything above the plant eater would be affected because the number of units of energy available has been reduced by the killing off of a number of the primary consumers. However, if birds do eat insects that have been killed or are dying because of the insecticides they become poisoned and various side effects show up; for example, poorly formed egg shells in birds.)

3. Many new homes that are built in areas that were formerly cornfields or pasture lands become infested with field mice. What is the reason for the invasion of these mice? (The

area taken up by the house supported many mice before the house was built. The mice still require food and because the surrounding area already has its supply of mice, these displaced mice seek food in the houses that rest on their territory.)

4. Why is controlled hunting in a deer range generally desirable by conservationists? (Without predators to keep the numbers of grazing deer within optimum numbers, the deer would not have sufficient green plant materials for all to survive. Many deer would die of starvation during the winter when food supply is at a premium.) NOTE: Man has been responsible for ridding the range of predators and also cutting down the area of the deer range.

5. Many lakes and ponds have hundreds of bream, sunfish, bluegill, perch, etc., but none of them are large enough to be considered as "catching size." What might be the reason for their small size? (One reason might be the fact that the lakes or ponds have large quantities of plant nutrients which cause a prolific growth of plants. This provides sufficient protection for these small fish to hide from their predators but since these small fish are not primary consumers they cannot get enough food in the lake or pond to grow larger but do find enough food to carry on their activity.)

6. Relate the exponential growth on one population in an energy pyramid with the size of other populations on the pyramid. What happens to those that prey on the growing population? What happens to those that are preyed upon? What are the outcomes of this destabilization of the food chain?

QUADS IN THE WORLD ROOM

PURPOSE:

(a) To show the distribution of people by continent
(b) To show the increase of population and where it is occuring most rapidly
(c) To show proportionally the energy consumption by continent
(d) To understand what energy demands will be placed upon this planet if population expands at present rates

MATERIALS: Discussions/activities, continent name tags, toothpicks or counters

PROCEDURE: NOTE: A quad is equal to one quadrillion Btu (1,000,000,000,000,000).

One British Thermal Unit (Btu) is the amount of energy needed to heat one pound of water to one degree F°.

This activity helps participating students experience growth and its effects. It is fun to do, but, at the same time, provides valuable insight into the relationship between population growth and finite energy resources.

ONE DAY AHEAD you will need to prepare Continent name tags. These will be distributed by lot on game day. Students should pin their tags to their clothing. You will need:

Continent Labels	No.
ASIA	12
EUROPE	4
AFRICA	2
LATIN AMERICA	2
NORTH AMERICA	1
REST OF THE WORLD	1

In the World Room, one student represents 200 million people. According to proportion we have worked out, you will start the World Room with 22 students. (That's 22 x 200 million - 4.4 billion, or approximately the present world population.) Using this formula, you will need eight more students to project the population growth for the year 1990. You will need a total of 43 students to project ahead just 40 years to A.D. 2020. (Most likely you will need to borrow some students from another class to act out the population growth.)

NOTE: You need 220 toothpicks, or counters. We will explain the purpose of these on the next page.

Begin by allowing 22 students into the room. The remaining students can either remain outside or occupy an out-of-the-way corner where the desks have been pushed. (Students who will not make up the total number of students eventually used in the activity can act as observers, and report observations about behavior, crowding, resource distribution, and the like.) Explain that each of the 22 students represents about 200 million people, and that all 22 represent the world's current population (4.4 billion).

Next, divide the 22 students by continent. Have the students who claimed the Asian tags go to one corner of the cleared space in the room. Have the students who represent Europe go to another unoccupied corner; those who represent Africa to another, and so on. The student representing North America should go to the center of the space.

Explain that the toothpicks you are about to distribute represent 220 quads (Q) of energy. (At this time you will need to explain what is meant by the concept of quad and Btu.) This amount represents the total world consumption of energy in 1975. According to United Nations Statistics, worldwide energy consumption in 1975 was 221 Q.

Distribute the 200 toothpicks according to the following formula:

CONTINENT	NUMBER OF ENERGY QUADS (TOOTHPICKS) EACH CONTINENT MEMBER RECEIVES
ASIA	37
EUROPE	92
AFRICA	3
LATIN AMERICA	12
NORTH AMERICA	73
REMAINING WORLD	3

Review with the students some general implications they might preceive from the relative grouping of students and the number of toothpicks held by each group.

Some Growth Projection Activities

1. How many toothpicks would each of the other four continents in the room have to receive to equal the current per capita standard of energy consumption of North America?

--The person representing North American consumes 73 Q.

--The 12 people representing Asia consume 37 Q or about 3 Q each (37 ÷ 12 = 3.1). Each person would need an increase of 70 Q to equal the per capita energy consumption of North America (73 - 3 = 70).

--The four people representing Europe consume 92 Q or about 23 Q each (92 ÷ 4 = 23). Each person would need an increase of 50 Q to equal the per capita energy consumption of North America (73 - 23 = 50).

--The two people representing Africa consume 3 Q or almost 2 Q each (3 ÷ 2 = 1.5). Each person would need an increase of 71 Q to equal the per capita energy consumption of North America (73 - 2 = 71).

--The two people representing Latin America consume 12 Q or 6 Q each (12 ÷ 2 = 6). Each person would need and increase of 67 Q to equal the per capita energy consumption of North America (73 - 6 = 67).

How much, then, would world production of energy have to increase in a year to meet this demand?

Asia	70 Q increase x 12 = 840
Europe	50 Q increase x 4 = 200
Africa	71 Q increase x 2 = 142
Latin America	67 Q increase x 2 = 134

TOTAL increase necessary in world energy production 1316 Q

2. Project the population in the year 1990. That is, add eight more students to the room. Add five to the Asian group, one to Africa, and two to Latin America. How crowded does the room seem? What new problems could occur as a result of the increase? Student answers will vary, but some likely responses are: psychological problems (frustration, anger) due to crowding, energy distribution and supply problems, dhortages of food, raw materials, etc.

3. Project population ahead to the year 2020. That is, add 13 more students to the room. Remember, you've doubled the population in just 40 years! How much more crowded does it seem? What problems may occur due to the increase? How do you feel about the increase?

4. How many more toothpicks would the room need to raise world energy consumption in 2020 to the current North American per capita consumption? How many times would annual energy production have to increase?

   8 people (question 2) + 13 people (question 3) = 21 people added to represent world population in 2020.

   21 x 73 (amount of present U.S. consumption) = 1533 Q needed for additional people.

   1533 + 1316 (additional Q needed by original population to reach present North American level of consumption) = 2849 additional toothpicks needed.

   Annual energy production would have to increase almost 13 times (2849 ÷ 220 Q present annual world energy consumption = 12.95).

Extending the Learning

(1) When is more too much? We suggest you focus on the effect of crowding by asking students how they could all work comfortably in a confined area. Point to a small area near a wall, bounded by desks on the other three sides. Tell students that they should move into this area and complete the assignment for the day, whatever it may be. Do not move desks or chairs into the area. Students will have to make do with whatever is in the area when they move into it. Next, ask yet another class to come to the classroom. Tell your class that because of forces beyond your control, they will have to take all the new members into the same confined area--and still complete the task for the day. Add members of the new class a few at a time so the jump in population is not too startling. Then keep increasing the population. Note the point at which students said they were uncomfortable and at what point they were unable to get their work done.

Conclude by discussing the effects of overcrowding on production and personal interactions. What would change if the class size increased by this much every day? What would be lost? What new resources would be needed? What other problems would arise from overcrowding?

Extending the Learning

(2) Below are some figures regarding the original 22 people in the World Room. We suggest making labels and having each student pin one on before entering the World Room. At the end of the activity, discuss these percentages and numbers.

of the 22 students who represent present world population:

Number of Students	Percent	Characteristics
18	86	are desperately poor
13	62	earn less than $200 per year
11	52	are women
8	38	are children
8	38	are illiterate
8	38	suffer from malnutrition
6	29	are Hindu, Buddhist, or members of 100 other Eastern religions
5	24	are Christian
3	14	are Muslim
1	1	have college educations

4500 years ago, there was only one person in the room (that is, about 200 million people in the world). Men were probably hunters. Women were probably food gatherers.

NSTA Exponential
Growth Unit

APPENDIX

Forgotten fundamentals of the energy crisis

Albert A. Bartlett
Department of Physics and Astrophysics, University of Colorado, Boulder, Colorado 80309

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"Facts do not cease to exist because they are ignored;" Aldous Huxley.

I. INTRODUCTION[1]

The energy crisis has been brought into focus by President Carter's message to the American people on April 18 and by his message to the Congress on April 20, 1977. Although the President spoke of the gravity of the energy situation when he said that it was "unprecedented in our history," his messages have triggered an avalanche of critical responses from national political and business leaders. A very common criticism of the President's message is that he failed to give sufficient emphasis to increased fuel production as a way of easing the crisis. The President proposed an escalating tax on gasoline and a tax on the large gas guzzling cars in order to reduce gasoline consumption. These taxes have been attacked by politicians, by labor leaders, and by the manufacturers of the "gas guzzlers" who convey the impression that one of the options that is open to us is to go ahead using gasoline as we have used it in the past.

We have the vague feeling that arctic oil from Alaska will greatly reduce our dependence on foreign oil. We have recently heard political leaders speaking of energy self-sufficiency for the U.S. and of "Project Independence." The divergent discussion of the energy problem creates confusion rather than clarity, and from the confusion many Americans draw the conclusion that the energy shortage is mainly a matter of manipulation or of interpretation. It then follows in the minds of many that the shortage can be "solved" by congressional action in the manner in which we "solve" social and political problems.

Many people seem comfortably confident that the problem is being dealt with by experts who understand it. However, when one sees the great hardships that people suffered in the northeastern U.S. in January 1977 because of the shortage of fossil fuels, one may begin to wonder about the long-range wisdom of the way that our society has developed.

What are the fundamentals of the energy crisis?

Rather than to travel into the sticky abyss of statistics it is better to rely on a few data and on the pristine simplicity of elementary mathematics. With these it is possible to gain a clear understanding of the origins, scope, and implications of the energy crisis.

II. BACKGROUND

When a quantity such as the rate of consumption of a resource (measured in tons per year or in barrels per year) is growing a fixed percent per year, the growth is said to be exponential. The important property of the growth is that the time required for the growing quantity to increase its size by a fixed fraction is constant. For example, a growth of 5% (a fixed fraction) per year (a constant time interval) is exponential. It follows that a constant time will be required for the growing quantity to double its size (increase by 100%). This time is called the *doubling time* T_2, and it is related to P, the percent growth per unit time by a very simple relation that should be a central part of the educational repertoire of every American.

$$T_2 = 70/P.$$

As an example, a growth rate rate of 5%/yr will result in the doubling of the size of the growing quantity in a time $T_2 = 70/5 = 14$ yr. In two doubling times (28 yr) the growing quantity will double twice (quadruple) in size. In three doubling times its size will increase eightfold ($2^3 = 8$); in four doubling times it will increase sixteenfold ($2^4 = 16$); etc. It is natural then to talk of growth in terms of powers of 2.

III. THE POWER OF POWERS OF TWO

Legend has it that the game of chess was invented by a mathematician who worked for an ancient king. As a reward for the invention the mathematician asked for the amount of wheat that would be determined by the following process: He asked the king to place 1 grain of wheat on the first square of the chess board, double this and put 2 grains on the second square, and continue this way, putting on each square twice the number of grains that were on the preceding square. The filling of the chessboard is shown in Table I. We see that on the last square one will place 2^{63} grains and the total number of grains on the board will then be one grain less than 2^{64}.

How much wheat is 2^{64} grains? Simple arithmetic shows that it is approximately 500 times the 1976 annual worldwide harvest of wheat! This amount is probably larger than all the wheat that has been harvested by humans in the history of the earth! How did we get to this enormous number? It is simple; we started with 1 grain of wheat and we doubled it a mere 63 times!

Exponential growth is characterized by doubling, and a few doublings can lead quickly to enormous numbers.

The example of the chessboard (Table I) shows us another important aspect of exponential growth; *the increase in any doubling is approximately equal to the sum of all the preceding growth!* Note that when 8 grains are placed on the 4th square, the 8 is greater than the total of 7 grains that were already on the board. The 32 grains placed on the 6th square are more than the total of 31 grains that were already on the board. Covering any square requires one grain more than the total number of grains that are already on the board.

Table I. Filling the squares on the chessboard.

Square Number	Grains On Square	Total grains Thus far
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
64	2^{63}	$2^{64}-1$

On April 18, 1977, President Carter told the American people,

"And in each of these decades (the 1950s and 1960s),

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> more oil was consumed than in all of man's previous history combined."

We can now see that this astounding observation is a simple consequence of a growth rate whose doubling time is $T_2 = 10$ yr (one decade). The growth rate which has this doubling time is $P = 70/10 = 7\%/\text{yr}$.

When we read that the demand for electrical power in the U.S. is expected to double in the next 10-12 yr we should recognize that this means that the quantity of electrical energy that will be used in these 10-12 yr will be approximately equal to the total of all of the electrical energy that has been used in the entire history of the electrical industry in this country! *Many people find it hard to believe that when the rate of consumption is growing a mere 7%/yr, the consumption in one decade exceeds the total of all of the previous consumption.*

Populations tend to grow exponentially. The world population in 1975 was estimated to be 4 billion people and it was growing at the rate of 1.9%/yr. It is easy to calculate that at this low rate of growth the world population would increase by one billion in less than 12 yr, the population would double in 36 yr, the population would grow to a density of 1 person/m^2 on the dry land surface of the earth (excluding Antarctica) in 550 yr, and the mass of people would equal the mass of the earth in a mere 1620 yr! Tiny growth rates can yield incredible numbers in modest periods of time! Since it is obvious that people could never live at the density of 1 person/m^2 over the land area of the earth, it is obvious that the earth will experience zero population growth. The present high birth rate and/or the present low death rate will change until they have the same numerical value, and this will probably happen in a time much shorter than 550 yr.

A recent report suggested that the rate of growth of world population had dropped from 1.9%/yr to 1.64%/yr.[2] Such a drop would certainly qualify as the best news the human race has ever had! The report seemed to suggest that the drop in this growth rate was evidence that the population crisis had passed, but it is easy to see that this is not the case. The arithmetic shows that an annual growth rate of 1.64% will do anything that an annual rate of 1.9% will do; it just takes a little longer. For example, the world population would increase by one billion people in 13.6 yr instead of in 11.7 yr.

Compound interest on an account in the savings bank causes the account balance to grow exponentially. One dollar at an interest rate of 5%/yr compounded continuously will grow in 500 yr to 72 billion dollars and the interest at the end of the 500 yr would be coming in at the magnificent rate of $114/s. If left untouched for another doubling time of 14 yr, the account balance would be 144 billion dollars and the interest would be accumulating at the rate of $228/s.

It is very useful to remember that steady exponential growth of n %/yr for a period of 70 yr (100 ln2) will produce growth by an overall factor of 2^n. Thus where the city of Boulder, Colorado, today has one overloaded sewer treatment plant, a steady population growth at the rate of 5%/yr would make it necessary in 70 yr (one human lifetime) to have $2^5 = 32$ overloaded sewer treatment plants!

Steady inflation causes prices to rise exponentially. An inflation rate of 6%/yr will, in 70 yr, cause prices to increase by a factor of 64! If the inflation continues at this rate, the $0.40 loaf of bread we feed our toddlers today will cost $25.60 when the toddlers are retired and living on their pensions!

It has even been proven that the number of miles of highway in the country tends to grow exponentially.[1(e),3]

The reader can suspect that the world's most important arithmetic is the arithmetic of the exponential function. *One can see that our long national history of population growth and of growth in our per-capita consumption of resources lie at the heart of our energy problem.*

IV. EXPONENTIAL GROWTH IN A FINITE ENVIRONMENT

Bacteria grow by division so that 1 bacterium becomes 2, the 2 divide to give 4, the 4 divide to give 8, etc. Consider a hypothetical strain of bacteria for which this division time is 1 min. The number of bacteria thus grows exponentially with a doubling time of 1 min. One bacterium is put in a bottle at 11:00 a.m. and it is observed that the bottle is full of bacteria at 12:00 noon. Here is a simple example of exponential growth in a finite environment. This is mathematically identical to the case of the exponentially growing consumption of our finite resources of fossil fuels. Keep this in mind as you ponder three questions about the bacteria:

(1) When was the bottle half-full? Answer: 11:59 a.m.!

(2) If you were an average bacterium in the bottle, at what time would you first realize that you were running out of space? Answer: There is no unique answer to this question, so let's ask, "At 11:55 a.m., when the bottle is only 3% filled (1/32) and is 97% open space (just yearning for development) would you perceive that there was a problem?" Some years ago someone wrote a letter to a Boulder newspaper to say that there was no problem with population growth in Boulder Valley. The reason given was that there was 15 times as much open space as had already been developed. When one thinks of the bacteria in the bottle one sees that the time in Boulder Valley is 4 min before noon! See Table II.

Table II. The last minutes in the bottle.

11:54 a.m.	1/64 full (1.5%)	63/64 empty
11:55 a.m.	1/32 full (3%)	31/32 empty
11:56 a.m.	1/16 full (6%)	15/16 empty
11:57 a.m.	1/8 full (12%)	7/8 empty
11:58 a.m.	1/4 full (25%)	3/4 empty
11:59 a.m.	1/2 full (50%)	1/2 empty
12 noon	full (100%)	empty

Suppose that at 11:58 a.m. some farsighted bacteria realize that they are running out of space and consequently, with a great expenditure of effort and funds, they launch a search for new bottles. They look offshore on the outer continental shelf and in the Arctic, and at 11:59 a.m. they discover three new empty bottles. Great sighs of relief come from all the worried bacteria, because this magnificent discovery is three times the number of bottles that had hitherto been known. The discovery quadruples the total space resource known to the bacteria. Surely this will solve the problem so that the bacteria can be self-sufficient in space. The bacterial "Project Independence" must now have achieved its goal.

(3) How long can the bacterial growth continue if the total space resources are quadrupled? Answer: Two more doubling times (minutes)! See Table III.

James Schlesinger, Secretary of Energy in President Carter's Cabinet recently noted that in the energy crisis "we have a classic case of exponential growth against a finite source."[4]

Table III. The effect of the discovery of three new bottles.

11:58 a.m.	Bottle No. 1 is one quarter full.
11:59 a.m.	Bottle No. 1 is half-full.
12:00 noon	Bottle No. 1 is full.
12:01 p.m.	Bottles No. 1 and 2 are both full.
12:02 p.m.	Bottles No. 1, 2, 3, 4 are all full.

Quadrupling the resource extends the life of the resource by only two doubling times! When consumption grows exponentially, enormous increases in resources are consumed in a very short time!

V. LENGTH OF LIFE OF A FINITE RESOURCE WHEN THE RATE OF CONSUMPTION IS GROWING EXPONENTIALLY

Physicists would tend to agree that the world's mineral resources are finite. The extent of the resources is only incompletely known, although knowledge about the extent of the remaining resources is growing very rapidly. The consumption of resources is generally growing exponentially, and we would like to have an idea of how long resources will last. Let us plot a graph of the rate of consumption $r(t)$ of a resource (in units such as tons/yr) as a function of time measured in years. The area under the curve in the interval between times $t = 0$ (the present, where the rate of consumption is r_0) and $t = T$ will be a measure of the total consumption C in tons of the resource in the time interval. We can find the time T_e at which the total consumption C is equal to the size R of the resource and this time will be an estimate of the expiration time of the resource.

Imagine that the rate of consumption of a resource grows at a constant rate until the last of the resource is consumed, whereupon the rate of consumption falls abruptly to zero. It is appropriate to examine this model because this constant exponential growth is an accurate reflection of the goals and aspirations of our economic system. Unending growth of our rates of production and consumption and of our Gross National Product is the central theme of our economy and it is regarded as disastrous when actual rates of growth fall below the planned rates. Thus it is relevant to calculate the life expectancy of a resource under conditions of constant rates of growth. Under these conditions the period of time necessary to consume the known reserves of a resource may be called the exponential expiration time (EET) of the resource. The EET is a function of the known size R of the resource, of the current rate of use r_0 of the resource, and of the fractional growth per unit time k of the rate of consumption of the resource. The expression for the EET is derived in the Appendix where it appears as Eq. (6). This equation is known to scholars who deal in resource problems[5] but there is little evidence that it is known or understood by the political, industrial, business, or labor leaders who deal in energy resources, who speak and write on the energy crisis and who take pains to emphasize how essential it is to our society to have continued uninterrupted growth in all parts of our economy. The equation for the EET has been called the best-kept scientific secret of the century.[6]

Table IV. United States crude oil (lower 48 states). Units are 10^9 barrels (1 barrel = 42 U.S. gal. = 158.98 l.).

Ultimate total production (Ref. 7)	190
Produced to 1972	96.6
Percent of ultimate total production produced to 1972 (Ref. 7)	50.8%
Annual production rate 1970	3.29

VI. HOW LONG WILL OUR FOSSIL FUELS LAST?

The question of how long our resources will last is perhaps the most important question that can be asked in a modern industrial society. Dr. M. King Hubbert, a geophysicist now retired from the United States Geological Survey, is a world authority on the estimation of energy resources and on the prediction of their patterns of discovery and depletion. Many of the data used here come from Hubbert's papers.[7-10] Several of the figures in this paper are redrawn from figures in his papers. These papers are required reading for anyone who wishes to understand the fundamentals and many of the details of the problem.

Let us examine the situation in regard to production of domestic crude oil in the U.S. Table IV gives the relevant data. *Note that since one-half of our domestic petroleum has already been consumed, the "petroleum time" in the U.S. is 1 min before noon!* Figure 1 shows the historical trend in domestic production (consumption) of crude oil. Note that from 1870 to about 1930 the rate of production of domestic crude oil increased exponentially at a rate of 8.27%/yr with a doubling time of 8.4 yr. If the growth in the rate of production stopped and the rate of production was held constant at the 1970 rate, the remaining U.S. oil would last only (190—96.6)/3.29 = 28 yr! We are currently importing one-half of the petroleum we use. If these imports were completely cut off and if there was no growth in the rate of domestic consumption above the 1970 rate, our domestic petroleum reserves would last only 14 yr! The vast shale oil deposits of Colorado and Wyoming represent an enormous resource. Hubbert reports that the oil recoverable under 1965 techniques is 80 × 10^9 barrels, and he quotes other higher estimates. In the preparation of Table V, the figure 103.4 × 10^9 barrels was used as the estimate of U.S. shale oil so that the reserves used in the calculation of column 4 would be *twice* those that were used in the calculation of column 3. This table makes it clear that *when consumption is rising exponentially, a doubling of the remaining resource results in only a small increase in the life expectancy of the resource.*

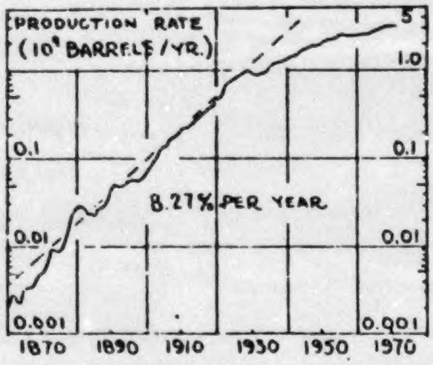

Fig. 1. History of U.S. crude oil production (semilogarithmic scale). Redrawn from Hubbert's Fig. 12, Ref. 7.

Table V. Exponential expiration time (EET) in years of various estimates of U.S. oil reserves for different rates of growth of annual production. Units are 10^9 barrels. This table is prepared by using Eq. (6) with $r_0 = 3.29 \times 10^9$ barrels/yr. *Note that this is domestic production which is only about one half of domestic consumption!* Column 1 is the percent annual growth rate. Column 2 is the lifetime (EET) of the resource which is calculated using $R = 190\text{-}96.6 = 93.4$ as the estimated oil remaining in the lower 48 states. Column 3 is the lifetime (EET) calculated $R = 93.4 + 10$ *to include the Alaskan oil.* Column 4 is the lifetime (EET) calculated using $R = 93.4 + 10 + 103.4 = 206.8$ to include Alaskan oil and a hypothetical estimate of U.S. oil shale.

Col. 1	Col. 2 (yr)	Col. 3 (yr)	Col. 4 (yr)
Zero	28.4	31.4	62.8
1%	25.0	27.3	48.8
2%	22.5	24.4	40.7
3%	20.5	22.1	35.3
4%	19.0	20.4	31.4
5%	17.7	18.9	28.4
6%	16.6	17.7	26.0
7%	15.6	16.6	24.1
8%	14.8	15.7	22.4
9%	14.1	14.9	21.1
10%	13.4	14.2	19.9

A reporter from CBS News, speaking about oil shale on a three-hour television special feature on energy (August 31, 1977) said

> "Most experts estimate that oil shale deposits like these near Rifle, Colorado, could provide more than a 100-yr supply."

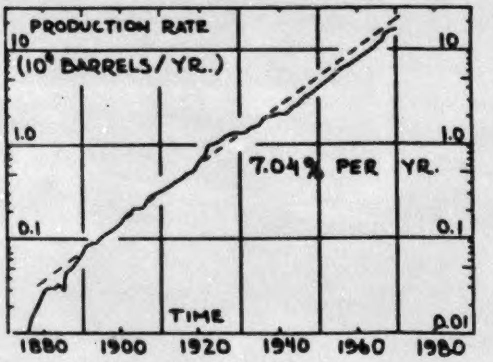

Fig. 2. History of world crude oil production (semilogarithmic scale). Redrawn from Hubbert's Fig. 6, Ref. 7.

Table VI. World crude oil data. Units are 10^9 barrels.

Ultimate total production (Ref. 7)	1952
Produced to 1972	261
Percent of total production produced to 1972 (Ref. 7)	13.4%
Annual production rate 1970	16.7

Note that a little more than 1/8 of the world's oil has been consumed. *The "world petroleum time" is between 2 and 3 min before noon, i.e. we are between 2 and 3 doubling times from the expiration of the resource.*

This statement should be compared with the figures given in column 4 of Table V. This comparison will serve to introduce the reader to the disturbing divergence between reassuring statements by authoritative sources and the results of simple calculations.

Anyone who wishes to talk about energy self-sufficiency for the United States (Project Independence) must understand Table V and the simple exponential calculations upon which it is based.

Table VI gives statistics on world production of crude oil. Figure 2 shows the historical trend in world crude oil production. Note that from 1890 to 1970 the production grew at a rate of 7.04%/yr, with a doubling time of 9.8 yr. It is easy to calculate that the world reserves of crude oil would last 101 yr if the growth in annual production was halted and production in the future was held constant at the 1970 level. Table VII shows the life expectancy (EET) of world crude oil reserves for various rates of growth of production and shows the amount by which the life expectancy is extended if one adds world deposits of oil shale. Column 4 is based on the assumption that the available shale oil is four times as large as the value reported by Hubbert. Note again that the effect of this very large hypothetical increase in the resource is very small. Figure 3 shows a dramatic graphical model from Mario Iona that can be used to represent this growth.[11] When consumption grows 7%/yr the consumption in any decade is approximately equal to the sum of all previous consumption as can be seen by the areas representing consumption in successive decades. The rectangle ABDC represents all the known oil, including all that has been used in the past, *and the rectangle CDFE represents the new discoveries that must be made if we wish the 7%/yr growth to continue one decade, from the year 2000 to 2010!*

Table VII. Life expectancy in years of various estimates of world oil reserves for different rates of growth of annual production. Units are 10^9 barrels. This table is prepared by using Eq. (6) with $r_0 = 16.7 \times 10^9$ barrels/yr. Column 1 is the percent annual growth rate of production. Column 2 is the EET of the resource calculated using $R = 1691$ as the estimate of the amount of the remaining oil. Column 3 is the EET calculated using $R = 1691 + 190 = 1881$ representing crude oil plus oil shale. Column 4 is the EET calculated using $R = 1691 + 4(190) = 2451$ which assumes that the amount of shale oil is four times the amount which is known now.

Col. 1	Col. 2 (yr)	Col. 3 (yr)	Col. 4 (yr)
Zero	101	113	147
1%	69.9	75.4	90.3
2%	55.3	59.0	68.5
3%	46.5	49.2	56.2
4%	40.5	42.6	48.2
5%	36.0	37.8	42.4
6%	32.6	34.1	38.0
7%	29.8	31.2	34.6
8%	27.6	28.8	31.8
9%	25.7	26.8	29.5
10%	24.1	25.1	27.5

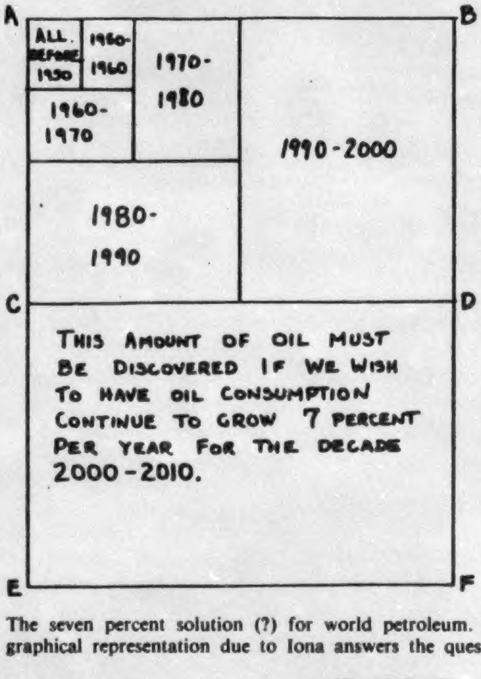
A
B
C
D
E
F
ALL BEFORE 1950
1950-1960
1960-1970
1970-1980
1980-1990
1990-2000
THIS AMOUNT OF OIL MUST BE DISCOVERED IF WE WISH TO HAVE OIL CONSUMPTION CONTINUE TO GROW 7 PERCENT PER YEAR FOR THE DECADE 2000-2010.

Fig. 3. The seven percent solution (?) for world petroleum. This graphical representation due to Iona answers the question,

"How long could world consumption of petroleum continue the 7% per year growth of Fig. 2?" The area of each rectangle represents the quantity of petroleum consumed in the labeled decades and the diagram makes it clear that when the doubling time is one decade, the quantity consumed in a decade is equal to the total of all preceding consumption. The area of the rectangle ABDC approximately represents the known world petroleum resource.

From these calculations we can draw a general conclusion of great importance. *When we are dealing with exponential growth we do not need to have an accurate estimate of the size of a resource in order to make a reliable estimate of how long the resource will last.*

A friend recently tried to reassure me by asserting that there remained undiscovered under our country at least as much oil as all we have ever used. Since it has been about 120 yr since the first discovery of oil in this country, he was sure that the undiscovered oil would be sufficient for another 120 yr. I had no success in convincing him that if such oil was found it would be sufficient only for one doubling time or about a decade.

As the reader ponders the seriousness of the situation and asks, "What will life be like without petroleum?" the thought arises of heating homes electrically or with solar power and of traveling in electric cars. A far more fundamental problem becomes apparent when one recognizes that modern agriculture is based on petroleum-powered machinery and on petroleum-based fertilizers. This is reflected in a definition of modern agriculture: "Modern agriculture is the use of land to convert petroleum into food."

Item

> "We have now reached the point in U.S. agriculture where we use 80 gallons of gasoline or its equivalent to raise an acre of corn, but only nine hours of human labor per crop acre for the average of all types of produce."[12]

Think for a moment of the effect of petroleum on American life. Petroleum has made it possible for American farms to be operated by only a tiny fraction of our population; only 1 American in 26 lived on a farm in 1976. The people thus displaced from our farms by petroleum-based mechanization have migrated to the cities where our ways of life are critically dependent on petroleum. The farms without the large number of people to do the work are also critically dependent on petroleum-based mechanization. The approaching exhaustion of the domestic reserves of petroleum and the rapid depletion of world reserves will have a profound effect on Americans in the cities and on the farms. It is clear that agriculture as we know it will experience major changes within the life expectancy of most of us, and with these changes could come a major further deterioration of world-wide levels of nutrition. The doubling time (36-42 yr) of world population (depending on whether the annual growth rate is 1.9% or 1.64%) means that we have this period of time in which we must double world food production if we wish to do no better than hold constant the fraction of the world population that is starving. This would mean that the number starving at the end of the doubling time would be twice the number that are starving today. This was put into bold relief by David Pimentel of Cornell University in an invited paper at the 1977 annual meeting of AAPT-APS (Chicago, 1977):

> "As a result of overpopulation and resource limitations, the world is fast losing its capacity to feed itself."
>
> "More alarming is the fact that while the world population doubled its numbers in about 30 years the world doubled its energy consumption within the past decade. Moreover, the use of energy in food production has been increasing faster than its use in many other sectors of the economy."

It is possible to calculate an absolute upper limit to the amount of crude oil the earth could contain. We simply assert that the volume of petroleum in the earth cannot be larger than the volume of the earth. The volume of the earth is 6.81×10^{21} barrels which would last for 4.1×10^{11} yr if the 1970 rate of consumption of oil held constant with no growth. The use of Eq. (6) shows that if the rate of consumption of petroleum continued on the growth curve of 7.04%/yr of Fig. 2, this earth full of oil will last only 342 yr!

It has frequently been suggested that coal will answer the U.S. and world energy needs for a long period in the future. What are the facts?

Table VIII. United States coal resource. Units are 10^9 metric tons.

Ultimate total production (Ref. 7)	
High estimate	1486
Low estimate	390
Produced through 1972 (My estimate from Hubbert's Fig. 22)	50
Percent of ultimate production produced through 1972	
Percent of high estimate	3%
Percent of low estimate	13%
Coal resource remaining	
High estimate	1436
Low estimate	340
Annual production rate, 1972	0.5
Rate of export of coal, 1974	0.06
Annual production rate, 1974	0.6
Annual production rate, 1976	0.665

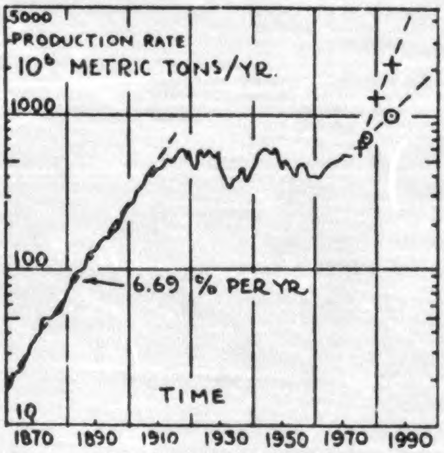

Fig. 4. History of U.S. coal production (semilogarithmic scale). Redrawn from Hubbert's Fig. 10, Ref. 7. In the upper right, the crosses in the steep dashed curve show the coal production goals of the Ford Administration, and the circles in the lower dashed curve show the production goals of the Carter Administration. From the close of the American Civil War to about the year 1910, coal production grew at a steady rate of 6.69%/yr. If this growth rate had continued undiminished after 1910, the small estimate of the size of U.S. coal reserves would have been consumed by about 1967 and the larger estimate of the size of these reserves would have been consumed by about the year 1990!

Table VIII shows data on U.S. coal production that are taken from several sources. Figure 4 shows the history of coal production in the U.S. Note that from 1860 to 1910, U.S. coal production grew exponentially at 6.69%/yr (T_2 = 10.4 yr). The production then leveled off at 0.5×10^9 tons/yr which held approximately constant until 1972 whereupon the rate started to rise steadily. Coal consumption remained level for 60 yr because our growing energy demands were met by petroleum and natural gas. In early 1976 the annual coal production goals of the U.S. government were 1.3 billion tons for 1980 and 2.1 billion tons for 1985. The 1976 production is now reported to have been 0.665 billion tons and the current goal is to raise annual production to a billion tons by 1985.[13] From these data we can see that the Ford administration's goals called for coal production to increase on the order of 10%/yr while the Carter administration is speaking of growth of production of approximately 5%/yr.

Table IX. Lifetime in years of United States coal (EET). The lifetime (EET) in years of U.S. coal reserves (both the high and low estimate of the U.S.G.S.) are shown for several rates of growth of production from the 1972 level of 0.5 ($\times 10^9$) metric tons per year.

	High Estimate (yr)	Low Estimate (yr)
Zero	2872	680
1%	339	205
2%	203	134
3%	149	102
4%	119	83
5%	99	71
6%	86	62
7%	76	55
8%	68	50
9%	62	46
10%	57	42
11%	52	39
12%	49	37
13%	46	35

Table IX shows the expiration times (EET) of the high and the low estimates of U.S. coal reserves for various rates of increase of the rate of production as calculated from the equation for the EET [Eq. (6)]. If we use the conservative smaller estimate of U.S. coal reserves we see that the growth of the rate of consumption will have to be held below 3%/yr if we want coal to last until our nation's tricentennial. If we want coal to last 200 yr, the rate of growth of annual consumption will have to be held below 1%/yr!

One obtains an interesting insight into the problem if one asks how long beyond the year 1910 could coal production have continued on the curve of exponential growth at the historic rate of 6.69%/yr of Fig. 4. The smaller estimate of U.S. coal would have been consumed around the year 1967 and the large estimate would have expired around the year 1990. *Thus it is clear that the use of coal as an energy source in 1978 and in the years to come is possible only because the growth in the annual production of coal was zero from 1910 to about 1972!*

VII. WHAT DO THE EXPERTS SAY?

Now that we have seen the facts let us compare them with statements from authoritative sources. Let us look first at a report to the Congress.

> "It is clear, particularly in the case of coal, that we have ample reserves."
> "We have an abundance of coal in the ground. Simply stated, the crux of the problem is how to get it out of the ground and use it in environmentally acceptable ways and on an economically competitive basis."
> "At current levels of output and recovery these reserves can be expected to last more than 500 years."[14]

Here is one of the most dangerous statements in the literature. It is dangerous because news media and the energy companies pick up the idea that "United States coal will last 500 years" while the media and the energy companies forget or ignore the important caveat with which the sentence began, "At current levels of output. . . ." The right-hand column of Table IX shows that at zero rate of growth of consumption even the low estimate of the U.S. coal resource "will last over 500 years." However, it is absolutely clear that the government does not plan to hold coal production constant "at current levels of output."

> "Coal reserves far exceed supplies of oil and gas, and yet coal supplies only 18% of our total energy. To maintain even this contribution we will need to increase coal production by 70% by 1985, but the real goal, to increase coal's share of the energy market will require a staggering growth rate."[15]

While the government is telling us that we must achieve enormous increases in the rate of coal production, other governmental officials are telling us that we can increase the rate of production of coal *and* have the resource last for a very long time.

> "The trillions of tons of coal lying under the United States will have to carry a large part of the nation's increased energy consumption, says (the) Director of the Energy Division of the Oak Ridge National Laboratories." "He estimated America's coal reserves are so huge, they could last 'a minimum of 300 years and probably a maximum of 1000 years.' "[16]

Compare the above statement of the life expectancy of U.S. coal reserves with the results of very simple calculations given in Table IX.

In the three-hour CBS television special on energy (August 31, 1977) a reporter stressed the great efforts that are being made to increase the rate of production of U.S. coal, and he summarized the situation in these words,

> "By the lowest estimate, we have enough (coal) for 200 years. By the highest, enough for more than a thousand years."

Again, compare the above statement with the results of simple calculations shown in Table IX.

While we read these news stories we are bombarded by advertisements by the energy companies which say that coal will last a long time at present rates of consumption *and* which say at the same time that we must dramatically increase our rate of production of coal.

> "At the rate the United States uses coal today, these reserves could help keep us in energy for the next two hundred years . . . Most coal used in America today is burned by electric power plants—(which)—consumed about 400 million tons of coal last year. By 1985 this figure could jump to nearly 700 million tons."[17]

Other advertisements stress just the 500 yr (no caveat).

> "We are sitting on half the world's known supply of coal —enough for over 500 years."[18]

Some ads stress the idea of self-sufficiency *without stating for how long a period we might be self-sufficient.*

> "Coal, the only fuel in which America is totally self-sufficient."[19]

Other ads suggest a deep lack of understanding of the fundamentals of the exponential function.

> "Yet today there are still those who shrill (sic) for less

> energy and no growth."
>
> "Now America is obligated to generate more energy—not less—merely to provide for its increasing population."
>
> "With oil and gas in short supply, where will that energy come from? Predominantly from coal. The U.S. Department of the Interior estimates America has 23% more coal than we dreamed of. 4,000,000,000,000 (trillion!) tons of it. Enough for over 500 years." (The non-sentences are in the original.)[20]

A simple calculation of the EET based on a current production rate of 0.6×10^9 tons/yr shows that the growth in the rate of production of coal can't exceed 0.8%/yr if the ad's 4×10^{12} tons of coal is to last for the ad's 500 yr. However, it should be noted that the 4×10^{12} tons cited in the ad is 2.8 times the size of the large estimate of U.S. coal reserves and is 12 times the size of the small estimate of U.S. coal reserves as cited by Hubbert.

When we view the range of creative information that is offered to the public we can not wonder that people are confused. We may *wish* that we could have rapid growth of the rate of consumption *and* have the reserves of U.S. coal last for a large number of years, but very simple calculations are all that is needed to prove that these two goals are incompatible. At this critical time in our nation's history we need to shift our faith to calculations (arithmetic) based on factual data and give up our belief in Walt Disney's First Law[21]: "Wishing will make it so."

On the broad aspects of the energy problem we note that the top executive of one of our great corporations is probably one of the world's authorities on the exponential growth of investments and compound interest. However, he observes that "the energy crisis was made in Washington." He ridicules "the modern-day occult prediction" of "computer print-outs" and warns against extrapolating past trends to estimate what may happen in the future. He then points out how American free-enterprise solved the great "Whale Oil Crisis" of the 1850s. With this single example as his data base he boldly extrapolates into the future to assure us that American ingenuity will solve the current energy crisis if the bureaucrats in Washington will only quit interfering.[22] It is encouraging to note that the person who made these statements in 1974, suggesting that the energy crisis was contrived rather than real, has now signed his name on an advertisement in *Newsweek Magazine* (Sept. 12, 1977) saying that

> "Energy is not a political issue. It's an issue of survival."
>
> "Time is running out."

However, the same issue of *Newsweek Magazine* carried two advertisements of coal which said

> "We've limited our use of coal while a supply that will last for centuries sits under our noses."
>
> "Coal—can provide our energy needs for centuries to come."

Carefully read this ad by the Edison Electric Institute for the Electric Companies telling us that

> "There is an increasing scarcity of certain *fuels*. But there is no scarcity of *energy*. There never *has* been. There never *will* be. There never *could* be. Energy is inexhaustible." (Emphasis is in the original.)[23]

We can read that a professor in a school of mining technology offers "proof" of the proposition

> "Mankind has the right to use the world's resources as it wishes, to the limits of its abilities. . . ."[24]

We have the opening sentence of a major scientific study of the energy problem,

> "The United States has an abundance of energy resources; fossil fuels (mostly coal and oil shale) adequate for centuries, fissionable nuclear fuels adequate for millenia and solar energy that will last indefinitely."[25]

We can read the words of an educated authority who asserts that there is no problem of shortages of resources,

> "It is not true that we are running out of resources that can be easily and cheaply exploited without regard for future operations."

His next sentence denies that growth is a serious component of the energy problem;

> "It is *not* true that we must turn our back on economic growth" (emphasis is in the original).

Three sentences later he says that there may be a problem,

> "We must face the fact that the well of nonrenewable natural resources is not bottomless."[26]

He does suggest that lack of "leadership" is part of the problem.

We have a statement by Ralph Nader,

> "The supply of oil, gas, and coal in this country is enormous and enough for hundreds of years. It is not a question of supply but a question of price and profits, of monopolies and undue political influence."[27]

Expert analysis of the problem can yield unusual recommendations. We have the opening paper in an energy conference in which a speaker from a major energy company makes no mention of the contribution of growth to the energy crisis when he asserts that,

> "The core of the energy problem both U.S. and worldwide" is "our excessive dependence on our two scarcest energy resources—oil and natural gas."

For him continued growth is not part of the problem, it is part of the solution!

> "*More* energy must be made available at a higher rate of growth than normal—in the neighborhood of 6 percent per year compared to a recent historical growth rate of 4 percent per year."[28]

The patient is suffering from cancer, and after a careful study the doctor prescribes the remedy; give the patient more cancer. Here is a second case where cancer is prescribed as the cure for cancer.

> "The National Petroleum Council in its report to the energy industry on the energy crisis" observed that "Restrictions on energy demand growth could prove (to be) expensive and undesirable. . . . The Council 'flatly rejected' any conservation-type measures proposing instead the production of more energy sources domestically and the easing of environmental controls."[29]

Study this statement carefully:

> "Energy industries agree that to achieve some form of energy self-sufficiency the U.S. must mine all the coal that it can."[30]

The plausibility of this statement disappears and its real meaning becomes apparent when we paraphrase it,

> "The more rapidly we consume our resources the more self-sufficient we will be."

David Brower has referred to this as the policy of *"Strength through Exhaustion."*[31] This policy has many powerful adherents. For example, on the three-hour CBS television special on energy (Aug. 31, 1977) William Simon, energy adviser to President Ford said

> We should be "trying to get as many holes drilled as possible to get the proven (oil) reserve. . . ."

Is it in the national interest to get and use these reserves as rapidly as possible?

We certainly get no sense of urgency from the remarks of the

Board Chairman of a major multinational energy corporation who concludes the discussion "Let's Talk Frankly About Energy" with his mild assessment of what we must do.

> "Getting on top of the energy problem won't be easy. It will be an expensive and time-consuming task. It will require courage, creativeness and discipline. . . ."[32]

If one searches beyond the work of Hubbert for an indication of others who understand the fundamental arithmetic of the problem one finds occasional encouraging evidence.[33] However, when one compares the results of the simple exponential calculations with news stories, with statements from public officials, and with assertions in advertisements of the energy companies it is hard to imagine that this arithmetic is widely understood.

The arithmetic of growth is the forgotten fundamental of the energy crisis.

VIII. A WORD OF CAUTION

We must note that these calculations of the EET of fossil fuels are not predictions of the future. They simply give us first-order estimates of the life expectancies of known quantities of several fuels under the conditions of steady growth which our society and our government hold sacred. These estimates are emphasized as aids to understanding the consequences of any particular growth scenario that the reader may want to consider or to evaluate.

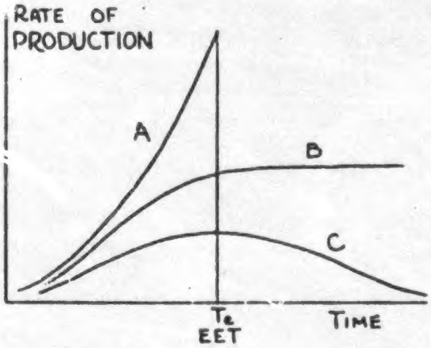

Fig. 5. Three patterns of growth. Curve A represents steady exponential growth in the rate of production of a non-renewable resource until the resource is exhausted at T_e, the exponential expiration time (EET). The area under the curve from the present ($t = 0$) to $t = T_e$ is equal to the known size of the resource. Curve C represents Hubbert's model of the way in which the rate of production of a nonrenewable resource rises and falls. This model is based on studies of the rate of use of resources which have been nearly completely consumed. The area under this curve from the present to $t = \infty$ is equal to the size of the resource. Curve B represents the rate of production of a renewable resource such as agricultural or forest products where a constant steady-state production can be maintained for long periods of time provided this production is not dependent on the use of a nonrenewable resource (such as petroleum) whose production is following a curve such as C.

The rate of production of our mineral resources will not rise exponentially until the EET is reached and then plunge abruptly to zero, as modeled in these calculations and as shown in curve A of Fig. 5 even though our national goals are predicated on uninterrupted growth. The rate of production of our nonrenewable mineral resources will not follow the classical *S*-shaped transition from an early period of exponential growth to a horizontal curve representing a constant rate of production, curve B. Such a curve can be achieved in the production of renewable resources such as food, forest products, or the production of solar energy, provided the rate of production of the renewable resource is not dependent on fossil fuels. Reference has already been made to the dependence of modern agriculture on petroleum, and as long as this dependence continues, the curve of agricultural production would be expected to follow curve C, (the curve for nonrenewable petroleum) rather than curve B. Although the rate of production of mineral resources has been growing exponentially one knows that at some time in the future the resource will be exhausted and the rate of production will return to zero. The past history, this one future datum and a careful study of the rate versus time of production of resources that have expired has led Dr. M. King Hubbert to the conclusion that the rate of production of a nonrenewable resource will rise and fall in the symmetrical manner of a Gaussian error curve as shown in curve C of Fig. 5. When he fits the data for U.S. oil production in the lower 48 states to a curve such as C, Hubbert finds that we are now just to the right of the peak. We have used one-half of the recoverable petroleum that was ever in the ground in the U.S. and in the future the rate of production can only go downhill. However, our national demand for petroleum has continued to grow exponentially and the difference between our demands and our production has been made up by imports. Bold initiatives by the Congress could temporarily reverse the trend and could put a small bump on the downhill side of the curve. Alaskan oil can put a little bump on the downhill side of the curve. The downhill trend on the right side of the curve was noted clearly by Deputy Energy Secretary John O'Leary under the headline, "U.S. Energy 'Disaster' Inevitable by 1985,"[34]

> "Although U.S. oil and gas production hit their peak several years ago and are declining by about 8 percent per year, O'Leary said, the nation has avoided serious problems by using more foreign oil." "We are walking into a disaster in the next three or four years with our eyes wide open."

The most dramatic conclusion that Hubbert draws from his curve for the complete cycle of U.S. oil production is that the consumption of the central 80% of the resource will take place in only 67 yr!

It is very sobering to face the downhill side of the curve and to note that in the past the rise in our annual *per capita* consumption of energy has gone hand-in-hand with the increase of our standard of living. It is more sobering to note the close coupling between our production of food and our use of petroleum. It is even more sobering to note that on March 7, 1956 (over 22 yr ago) Dr. Hubbert, addressing the conference in San Antonio, Texas, of a large group of petroleum engineers and geologists said

> "According to the best currently available information, the production of petroleum and natural gas on a world scale will probably pass its climax within the order of half a century, while for both the United States and for Texas, the peaks of production can be expected to occur within the next 10 or 15 years." (i.e., between 1966 and 1971).

Pazik[33] tells of the shock this statement and the related analysis caused in oil industry circles and he tells about the efforts that were made by the "experts" to ignore this and the other results of the analysis made by Hubbert.

IX. WHAT DO WE DO NOW?

The problems are such that we have rather few options. All of the following points are vital:

(i) We must educate all of our people to an understanding of the arithmetic and consequences of growth, especially in terms of the earth's finite resources. David Brower has observed that

> "The promotion of growth is simply a sophisticated way to steal from our children."

(ii) We must educate people to the critical urgency of abandoning our religious belief in the disastrous dogma that "growth is good," that "bigger is better," that "we must grow or we will stagnate," etc., etc. We must realize that growth is but an adolescent phase of life which stops when physical maturity is reached. If growth continues in the period of maturity it is called obesity or cancer. Prescribing growth as the cure for the energy crisis[28-29] has all the logic of prescribing increasing quantities of food as a remedy for obesity. *The recent occasion of our nation's 200th anniversary would be an appropriate time to make the transition from national adolescence to national maturity.*

(iii) We must conserve in the use and consumption of everything. We must outlaw planned obsolescence. We must recognize that, as important as it is to conserve, the arithmetic shows clearly that large savings from conservation will be wiped out in short times by even modest rates of growth. For example, in a dozen or two years a massive federal program might result in one-half of the heat for the buildings where we live and work being supplied by solar energy instead of by fossil fuels. This would save 10% of our national use of fossil fuels, but this enormous saving could be completely wiped out by two years of 5% growth. Conservation alone cannot do the job! The most effective way to conserve is to stop the growth in consumption.

As we consider the absolute urgency of conservation we must recognize that some powerful people are hostile to the concept of conservation. One of our great multinational oil companies has advertised that conservation is

> "good for you—but not if there's too much"

and in the same ad they noted that

> "Conservation does no harm."[33]

In his message to the American people President Carter proposed a tax on large "gas guzzling" cars. General Motors Chairman Thomas Murphy had the following reaction to this proposal to conserve energy:

> "Murphy calls the excise tax on big cars, coupled with rebates on small cars 'one of the most simplistic irresponsible and short-sighted ideas ever conceived by the hip-shooting marketeers of the Potomac.' "[34]

Big labor is hostile to this same conservation measure. Leonard Woodcock, President of the United Auto Workers said of the tax:

> "I respectfully suggest that the proposal is wrong." "It is not properly thought through and should be withdrawn."[37]

Congress is not enthusiastic about conservation.

> "Look for Senate leaders on both sides of the aisle—including Chairman Russell Long of the Finance Committee and Minority Leader Howard Baker—to gang up on Carter's energy package. The two influential lawmakers want more stress on the production of oil, not so much on conservation."[38]

Closer to home we can note that our governors don't show much enthusiasm for conservation.

> "The nation's governors told President Carter that the federal government is placing too much emphasis on conservation and not enough on developing new resources."[39]

With all this influential opposition one can see how difficult it will be to launch major national programs of energy conservation.

(iv) We must recycle almost everything. Except for the continuous input of sunlight the human race must finish the trip with the supplies that were aboard when the "spaceship earth" was launched.

(v) We must invest great sums in research (a) to develop the use of solar, geothermal, wind, tidal, biomass, and alternative energy sources; (b) to reduce the problems of nuclear fission power plants; (c) to explore the possibility that we may be able to harness nuclear fusion. These investments must not be made with the idea that if these research programs are successful the new energy sources could sustain growth for a few more doubling times. The investments must be made with the goal that the new energy sources could take over the energy load in a mature and stable society in which fossil fuels are used on a declining exponential curve as chemical raw materials and are not used as fuel for combustion. One great area of responsibility of our community of scientists and engineers is vigorous pursuit of research and development in all these areas. These areas offer great opportunity to creative young people.

Perhaps the most critical things that we must do is to decentralize, and consequently humanize, the scale and scope of our national industrial and utility enterprises.[40]

(vi) We must recognize that it is exceedingly unscientific to promote ever-increasing rates of consumption of our fuel resources based on complete confidence that science, technology, and the economics of the marketplace will combine to produce vast new energy resources as they are needed. Note the certainty that characterizes this confidence.

> "Coal could help fight a rear-guard action to provide time for scientific breakthroughs which will move the world from the fossil fuel era of wood, gas, oil, and coal to the perpetual energy era of infinitely renewable energy resources."[41]
>
> "The supply (of coal) is adequate to carry the U.S. well past the transition from the end of the oil and gas era to new, possibly not discovered sources of energy in the 2000s."[42]

There seems to be an almost complete absence of the caution that would counsel us to stop the growth of our national energy appetite until these "unlimited energy resources" are proven to be capable of carrying the national energy load. We must recognize that it is not acceptable to base our national future on the motto "When in doubt, gamble."

Fusion is most commonly mentioned as being an unlimited energy source. The optimism that leads some people to believe that fusion power will be ready whenever it is needed should be balanced against this opening statement in a report on fusion from MIT.

> "Designing a fusion reactor in 1977 is a little like planning to reach heaven: theories abound on how to do it, and many people are trying, but no one alive has ever succeeded."[43]

If the generation of electric power from fusion was achieved today, we could ask how long would it then be before fusion could play a significant role in our national energy picture. The time-constant for the replacement of one major energy source by another can be estimated from the fact that the first nuclear fission reactor was operated in December 1942. Even though the recent growth of nuclear energy in the U.S. has been spectacular, it was not until around 1972 that that annual nuclear energy consumption equaled our annual energy consumption from firewood! By 1973 nuclear energy had climbed to the point

where it supplied 1.3% of our U.S. total annual energy consumption and 4.6% of our electrical power.[44]
Thus in 31 yr nuclear energy has grown to provide only a small fraction of our energy needs. Had there been no growth of our national electrical needs since 1942, today's nuclear plants would be supplying 41% of our national electrical power.

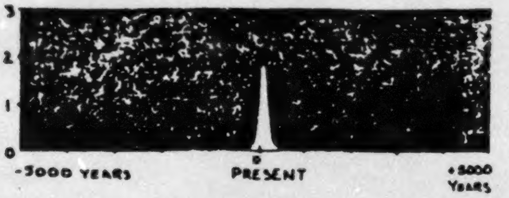

Fig. 6. The delta function in the darkness. Redrawn from Hubbert's Fig. 69. Ref. 7. The epoch of the world's use of its fossil fuels is shown on a time scale of human history from 5000 yr ago to 5000 yr in the future. The vertical axis is the rate of consumption of fossil fuels measured in units of 10^{14}kW h/yr. The vertical scale is a linear scale.

(vii) We can no longer sit back and deplore the lack of "leadership" and the lack of response of our political system. In the immortal words of Pogo, "We have met the enemy, and they's us." We are the leaders, we are vital parts of the political system and we have an enormous responsibility.

The arithmetic makes clear what will happen if we hope that we can continue to increase our rate of consumption of fossil fuels. Some experts suggest that the system will take care of itself and that growth will stop naturally, even though they know that cancer, if left to run its natural course, always stops when the host is consumed. My seven suggestions are offered in the spirit of *preventive* medicine.

X. CONCLUSION

The preceding calculations are offered as guideposts which must be understood by those who would deal constructively with the energy crisis. The role and limitations of science in analyzing and in solving our problems was beautifully expressed by Gustav Lebon (1841-1931).

> "Science has promised us truth; an understanding of such relationships as our minds can grasp. It has never promised us either peace or happiness."

Perhaps the most succinct conclusion that is indicated by the analysis above is taken from the immortal words of Pogo, "The future ain't what it used to be!" The American system of free enterprise has flourished for 200 yr with spectacular achievements. Until recently it flourished in a world whose energy resources were essentially infinite. Whenever one fossil fuel came into short supply, another could always be found to take its place. We are now close enough that we can see the end of the world's total supply of fossil fuels. The challenge that we must meet is set forth clearly in the question, "Can free enterprise survive in a finite world?" President Carter observed (April 18, 1977) that

> "If we fail to act soon we will face an economic, social, and political crisis that will threaten our free institutions." (See Fig. 6).

XI. A POSTSCRIPT FOR SCIENCE TEACHERS

For decades physics teachers throughout the world have discussed the *RC* circuit and the decay of radioactive atoms and have thus introduced the simple differential equation that gives rise to exponential decay of the charge on the capacitor or of the number of remaining radioactive nuclei. These provide a wonderful opportunity for us to digress and to point out that exponential arithmetic has great value outside of these two special examples in physics and to show our students that exponential arithmetic is probably the most important mathematics they will ever see. It is especially important for students to see how the change in the sign of the exponent can make an enormous difference in the behavior of the function. But we will need to do more. We must integrate the study of energy and of the exponential arithmetic into our courses as has been done, for example, in one new text.[44] In addition, we have an even larger task. As science teachers we have the great responsibility of participating constructively in the debates on growth and energy. We must be prepared to recognize opinions such as the following, which was expressed in a letter to me that was written by an ardent advocate of "controlled growth" on our local community.

> "I take no exception to your arguments regarding exponential growth."
>
> "I don't think the exponential argument is valid on the local level."

We must bring to these debates the realism of arithmetic and the new concept of precision in the use of language. We must convey to our students the urgency of analyzing all that they read for realism and precision. We must convey to our students the importance of making this analysis even though they are reading the works of an eminent national figure who is writing in one of the world's most widely circulated magazines. (The emphasis in the following quotation is in the original.)

> "The simple truth is that America has an abundance of energy resources."
>
> "An estimated 920 *trillion* cubic feet of natural gas still lies beneath the United States. Even at present consumption rates, this should last at least 45 years."
>
> "About 160 *billion* barrels of oil still lie below native ground or offshore. That's enough to last us into the next century at present rates of consumption."[45]

When students analyze these statements they can see that the first statement is false if "abundance" means "sufficient to continue currently accepted patterns of growth of rates consumption for as long as one or two human lifetimes." An evaluation of the second and third statements shows that they are falsely reassuring because they suggest the length of time our resources will last under the special condition of no growth of the rates of use of these resources. The condition of no growth in these rates is absolutely contrary to the precepts of our national worship of growth. It is completely misleading to introduce the results of "no growth" unless one is advocating "no growth."

If it is true that our natural gas reserves will last 45 yr at present rates of consumption (R/r_0 = 45 yr), then Eq. (6) shows that this amount of gas would last only 23.6 yr at an annual growth rate of 5%/yr, and only 17 yr at an annual growth rate of 10%/yr. When the third statement is analyzed one sees that the given figure of 160×10^9 barrels of reserves is roughly 60% larger than Hubbert's estimate. This amount would last 49 yr if oil was produced at the 1970 rate of 3.3×10^9 barrels/yr, held constant with no growth. However, our domestic consumption is now roughly twice the rate of domestic production, so *this amount of oil would satisfy domestic needs for only about 25 yr* *if there was no growth in these domestic needs.* If $R/r_0 = 25$ yr, then Eq. (6) shows that this amount of oil would last only 16.2 yr if production grew 5%/yr and only 12.5 yr if it grew 10%/yr.

We can conclude that the author is probably advocating growth in the *rate* at which we use fossil fuels from the following imprecise statement, "The fact is that we must produce more energy." Therefore the author's statements about the life expectancy of resources at current rates of use are irrelevant. When they are offered as reassurance of the lack of severity of our energy problem they are dangerously and irresponsibly misleading.

Students should be able to evaluate the same author's statement about coal,

> "At least 220 *billion* tons of immediately recoverable coal—awaits mining in the United States." This "could supply our energy needs for several centuries."

Students can see that the size of the coal reserves given by the author is significantly smaller than either of the two estimates given by Hubbert. They can see that it is imprecise and meaningless to suggest how long a resource will last if one says nothing about the rate of growth of production.

In addition to encouraging our students to carry out their responsibility to analyze what they read, we must encourage them to recognize the callous (and probably careless) inhumanity of a prominent person who is perhaps in his fifties,[43] offering reassurance to younger readers to the effect, "don't worry, we have enough petroleum to last into the next century." The writer is saying that "There is no need for *you* to worry, for there is enough petroleum for the rest of *my* life." Can we accept the urgings of those who advocate unending expansion and growth in the rates of consumption of our fossil fuel resources and who say "Why worry, we have enough to last into the next century."

We must give our students an appreciation of the critical urgency of evaluating the vague, imprecise, and meaningless statements that characterize so much of the public debate on the energy problem. The great benefits of the free press place on each individual the awesome responsibility of evaluating the things that he or she reads. Students of science and engineering have special responsibilities in the energy debate because the problems are quantitative and therefore many of the questions can be evaluated by simple analysis.

Students must be alert not only to the writings in the popular press but to the writings in college textbooks. In the bookstore of a school of engineering I purchased a book that was listed for one of the courses, possibly in political science. Here are a few interesting statements from the book[46]:

> "Our population is not growing too rapidly, but much too slowly."
>
> "To approach the problem ('the population scare') from the standpoint of numbers *per se* is to get the whole thing hopelessly backward."
>
> "Our coal supply alone, for example, is sufficient to power our economy for anywhere for 300 to 900 years—depending on the uses to which it is put—while gas and oil and coal together are obviously good for many centuries."
>
> "So whatever the *longterm* outlook for these energy sources, it is obvious (that) natural shortage cannot account for the *present* energy crunch."

Dr. Hubbert, speaking recently, noted that we do not have an energy crisis, we have an energy shortage. He then observed that *the energy shortage has produced a cultural crisis.* (See Fig. 7.)

We must emphasize to our students that they have a very special role in our society, a role that follows directly from their analytical abilities. It is their responsibility (and ours) to become the great humanists.

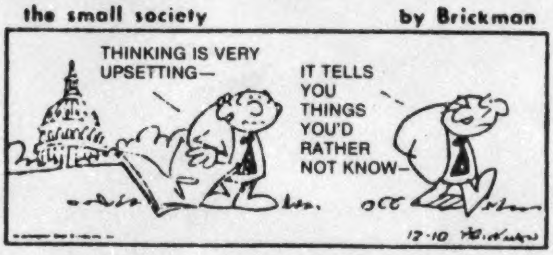
the small society
by Brickman
THINKING IS VERY UPSETTING—
IT TELLS YOU THINGS YOU'D RATHER NOT KNOW—
12-10

Fig. 7. Cartoon reproduced with the permission of the Washington Star Syndicate, Inc.

Note added in proof

Two incredible misrepresentations of the life expectancy of U.S. coal reserves have been called to my attention recently. *Time* (April 17, 1978, p. 74) said that "Beneath the pit heads of Appalachia and the Ohio Valley, and under the sprawling strip mines of the West, lie coal seams rich enough to meet the country's power needs for centuries, *no matter how much energy consumption may grow.*" (emphasis added) In reply to my letter correcting this, *Time* justified their statement by saying that they were using the Citibank estimate of U.S. coal reserves which is larger than the estimate used by Hubbert.

A beautiful booklet, "Energy and Economic Independence" (Energy Fuels Corporation of Denver, Denver, 1976) said, "As reported by *Forbes* magazine, the United States holds 437 billion tons of known (coal) reserves. That is equivalent to 1.8 trillion barrels of oil in British Thermal units, or *enough energy to keep 100 million large electric generating plants going for the next 800 years or so.*" (emphasis added) This is an accurate quotation from *Forbes*, the respected business magazine (December 15, 1975, p. 28) Long division is all that is needed to show that 437×10^9 tons of coal would supply our 1976 production of 0.665×10^9 tons per year for only 657 years, and we probably have fewer than 500 large electric generating plants in the U.S. today. This booklet concluded, "Your understanding of the facts about 'energy and economic independence' issue is of great importance."

A very thoughtful comment on fusion was made to me recently by a person who observed that it might prove to be the worst thing that ever happened to us if we succeed in using nuclear fusion to generate electrical energy because this success would lead us to conclude that we could continue the unrestrained growth in our annual energy consumption to the point (in a relatively few doubling times) where our energy production from the unlimited fusion resource was an appreciable fraction of the solar power input to the earth. This could have catastrophic consequences.

Richard Stout, columnist for the *New Republic*, noted (*Time*, March 27, 1978, p. 83) that in America, "We consume one third of all the energy, one third of the food and enjoy one half of the world's income. Can a disparity like this last? I think that much of the news in the next 50 years is going to turn on whether we yield to the inevitable graciously or vindictively."

ACKNOWLEDGMENTS

A great deal of correspondence and hundreds of conversations with dozens of people over six years have yielded many ideas, suggestions, and facts which I have incorporated here. I offer my sincere thanks to all who have helped.

APPENDIX

When a quantity such as the rate $r(t)$ of consumption of a resource grows a fixed percent per year, the growth is exponential.

$$r(t) = r_0 e^{kt} = r_0 2^{t/T_2}, \tag{1}$$

where r_0 is the current rate of consumption at $t = 0$, e is the base of natural logarithms, k is the fractional growth per year, and t is the time in years. The growing quantity will increase to twice its initial size in the doubling time T_2 where

$$T_2(\text{yr}) = (\ln 2)/k \approx 70/P, \tag{2}$$

and where P, the percent growth per year, is $100k$. The total consumption of a resource between the present ($t = 0$) and a future time T is

$$C = \int_0^T r(t)\, dt. \tag{3}$$

The consumption in a steady period of growth is

$$\begin{aligned} C &= r_0 \int_0^T e^{kt}\, dt \\ &= (r_0/k)(e^{kT} - 1). \end{aligned} \tag{4}$$

If the known size of the resource is R tons, then we can determine the exponential expiration time (EET) by finding the time T_e at which the total consumption C is equal to R:

$$R = (r_0/k)(e^{kT_e} - 1). \tag{5}$$

We may solve this for the exponential expiration time T_e,

$$\text{EET} = T_e = (1/k)\ln(kR/r_0 + 1). \tag{6}$$

This equation is valid for all positive values of k and for those negative values of k for which the argument of the logarithm is positive.

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[1] This paper is based on a series of articles, "The Exponential Function" which is appearing in "The Physics Teacher," (a) Phys. Teach. **14**, 393 (Oct. 1976); (b) **14**, 485 (Nov. 1976); (c) **15**, 37 (Jan. 1977); (d) **15**, 98 (Mar. 1977); (c) **15**, 225 (Apr. 1977); (f) **16**, 23 (Jan. 1978); (g) **16**, 92 (Feb. 1978); (h) **16**, 158 (Mar. 1978). An early version of this paper was presented at the Third Annual UMR-MEC Conference on Energy, held at the University of Missouri at Rolla, Oct. 12, 14, 1976, and appears in the volume of the Proceedings of the Conference. The early version, or minor revisions of it have been published in "Not Man Apart" published by Friends of the Earth, July/Aug. 1977, Vol. 7, No. 14, pp. 12-13; *The Vermillion Flycatcher* (Tucson Arizona Audubon Society, December 1977); *The Colorado Business Review* (Graduate School of Business Administration of the University of Colorado, Jan./Feb. 1978).

[2] Newsweek, Dec. 6, 1976.

[3] A. A. Bartlett, Civil Engin., Dec. 1969, p. 71.

[4] Time, April 25, 1977, p. 27.

[5] W. von Engelhardt, J. Goguel, M. King Hubbert, J. E. Prentice, R. A. Price, and R. Trumpy/Environ. Geol. **1**, 193-206 (1976).

[6] A. A. Bartlett, Proceedings of the Third Annual UMR-MEC Conference on Energy, University of Missouri at Rolla, Missouri, October 12-14, 1976, p. 10.

[7] "U.S. Energy Resources, a Review as of 1972," a background paper prepared at the request of the Hon. Henry M. Jackson, Chairman of the Committee on Interior and Insular Affairs of the United States Senate, pursuant to Senate Resolution 45. M. King Hubbert, *A National Fuels and Energy Policy Study, Serial 93-40 (92-75) Part I* (U.S. GPO, Washington, D.C., 1973), $2.35, 267 pages. This document is an invaluable source of data on consumption rates and trends in consumption, for both the U.S.A. and the world. In it Hubbert also sets forth the simple calculus of his methods of analysis. He does not confine his attention solely to exponential growth. He predicts that the rate of rise and subsequent fall of consumption of a resource will follow a symmetrical curve that looks like the normal error curve. Several figures in this paper are redrawn from Hubbert's paper.

[8] L. Ruedisili and M. Firebaugh, *Perspectives on Energy* (Oxford U.P., New York, 1975). This book contains a chapter by M. K. Hubbert, "Survey of World Energy Resources," pp. 92-122.

[9] National Academy of Science and National Research Council, *Resources and Man*, by M. King Hubbert (Freeman, San Francisco, 1969), Chap. 8.

[10] M. King Hubbert, "Energy Resources of the Earth," Sci. Am. Sept. 1971, p. 60. Reprinted as a book (Freeman, San Francisco, 1971).

[11] M. Iona, Phys. Teach. **15**, 324 (1977).

[12] Emile Benoit, "The Coming Age of Shortages," Bull. At. Sci., January 1976, p. 7. Benoit attributes his information to David Pimintel *et al.*, "Food Production and the Energy Crisis," Science **182**, 448 (Nov. 2, 1973). This article is the first of three by Benoir (Jan., Feb., Mar., 1976, Bull. At. Sci.) which are one of the best presentations I have read of coming problems of food, fuels, and resources.

[13] Newsweek, Jan. 31, 1977.

[14] "Factors Affecting the Use of Coal in Present and Future Energy Markets" a background paper prepared by The Congressional Research Service at the request of Sen. Henry M. Jackson, Chairman of the Committee on Interior and Insular Affairs of the United States Senate pursuant to Senate Resolution 45, a *National Fuels and Energy Policy Study Serial No. 93-9 (92-44)* (U.S. GPO, Washington, D.C., 1973), pp. 41, 42, 15.

[15] "The Energy Crisis" a booklet by the U.S. Energy Research and Development Agency (ERDA) Oak Ridge, Tennessee, no date, p. 3. (1975 or 1976).

[16] Associated Press story "Energy Head Stresses Coal Reserves," in the Boulder Daily Camera, July 5, 1975.

[17] "America's Coal: A Gold Mine of Energy" Exxon Corporation two-page full-color ad in Newsweek, 1975.

[18] "They're trying to tell us something. We're foolish not to listen" American Electric Power Company, Inc. Two-page ad in Newsweek, 1975.

[19] "The call to greater energy independence" American Electric Power Company, Inc., ad in Newsweek, Nov. 3, 1975.

[20] "An open letter on energy to those who are still employed." American Electric Power Company, Inc., ad in Newsweek, Jan. 12, 1976.

[21] W. H. Miernyk, Journal of Energy and Development **1**, No. 2, 223 (1976).

[22] "The Whale Oil, Chicken, and Energy Syndrome," an address before the Economic Club of Detroit by Walter B. Wriston, Chairman, First National City Corporation, Feb. 25, 1974.

[23] "The Transitional Storm, Part I, An Explanation," by the Edison Electric Institute for the Electric Companies, in "Broadcasting," July 26, 1976.

[24] "Moral Basis for Mineral Resource Use and Development Policy" by Charles O. Frush, "The Mines Magazine" Colorado School of Mines, March 1973, p. 20.

[25] J. C. Fisher, *Physics and the Energy Problem* (AIP, New York, 1974).

[26] "Opening Remarks, UMR-MEC Conference on Energy," R. L. Bisplinghoff. Proceedings of the Conference, Oct. 7-9, 1975, University of Missouri at Rolla.

[27] Washington Star, Feb. 12, 1977.

[28] L. G. Hauser, "Creating the Electric Energy Economy," Proceedings of the Second Annual UMR-MEC Conference on Energy, October 7-9, 1975, p. 3. University of Missouri at Rolla.

[29] Gil Bailey, "Conservation—Development Proposed As Solution," Washington Bureau of the Boulder Daily Camera, March 13, 1973.

[30] Time Mag., May 19, 1975, p. 55.

[31] D. Brower, "Not Man Alone" Vol. 6, No. 20, Nov. 1976; Friends of the Earth, 529 Commercial, San Francisco.

[32] C. C. Garvin, Jr., Chairman of the Exxon Corporation. Full page ad in the Rocky Mountain News, July 23, 1976.

[33] G. Pazik, in a special editorial feature, "Our Petroleum Predicament," in *Fishing Facts* ("The magazine for today's freshwater fisherman") Northwoods Publishing Co., P.O. Box 609, Menomonee Falls, WI 53051. Nov. 1976. Reprints are available at $0.30 each from the publisher. This is an excellent summary of the present situation and of the way we got into our petroleum predicament.

[34] The Arizona Republic, Feb. 8, 1978.

[35] "Conservation is like Cholesterol" an ad copyrighted 1976 by the Mobil Oil Corporation.

[36] Boulder Daily Camera, April 4, 1977.

[37] Boulder Daily Camera, May 16, 1977.

[38] U.S. News and World Report, July 25, 1977, p. 8.

[39] Boulder Daily Camera, July 10, 1977.

[40] Amory Lovins, "Energy Strategy, the Road Not Taken," Foreign Affairs, Oct. 1976. This material is now available as a book, *Soft Energy Paths: Toward a Durable Peace,* Ballinger, Cambridge, MA, 1977). It is said that this book "could very well be the most important book on energy policy of this decade."

[41] W. L. Rogers, Special Assistant *to the Secretary of the Interior, quoted in the* Denver Post, Nov. 19, 1976.

[42] Time Mag., April 4, 1977, p. 63.

[43] Technol. Rev., Dec. 1976, p. 21, reprinted in the second edition of Ref. 8.

[44] Robert H. Romer, *Energy—An Introduction to Physics* (Freeman, San Francisco, 1976), pp. 594-597. In addition to making energy the central theme of an introductory text, this book has 18 appendices (61 pages) of data ranging from "Units and conversion factors" to the "History of energy production and consumption in the world and in the United States" to "Exponential growth" to "Consumer prices of common sources of energy." The book is at once a text and a valuable source of reference data.

[45] *Melvin Laird, "The Energy Crisis: Made in U.S.A." Reader's Digest. Sept.* 1977, p. 56.

[46] M. Stanton Evans, *Clear and Present Dangers* (Harcourt Brace Jovanovich, New York, 1975).